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THE INTERNAL COMBUSTION ENGINE

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THE INTERNAL COMBUSTION ENGINE

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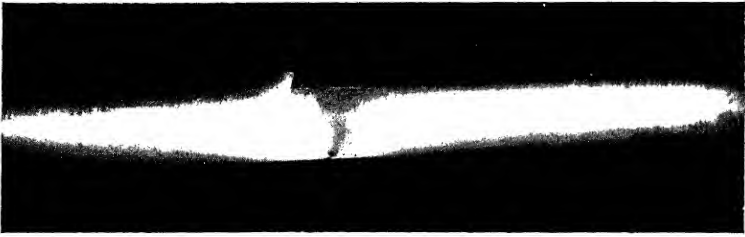
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FIGS. 1 and 2. Aluminium crystal in round bar at point of fracture

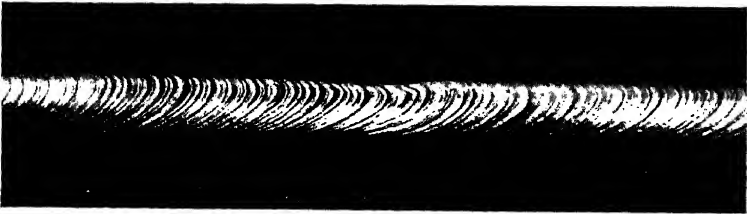


FIG. 3. Extended copper-aluminium crystal

FIG. 4. Extended brass crystal

DISTORTION
OF
METAL CRYSTALS

BY
C. F. ELAM
(MRS. G. H. TIPPER)

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PREFACE

DUCTILITY and great mechanical strength are two of the characteristic properties which have made metals so important industrially. Metallurgical literature contains much valuable information relating to plastic deformation. This is well known to many of those connected with the industry and metallurgical research, but the information is not so easily obtained by workers in other branches of science. Moreover, the development of the study of large metal crystals, and the application of X-ray analysis to their investigation, have introduced new methods of research which are still unfamiliar to many metallurgists. An attempt has been made in this book to gather together the results of the most important investigations relating to certain aspects of the distortion of metals, and then to see how far the facts lead us to a true understanding of the problem. It was decided to limit the scope of the book to an account of the changes in structure accompanying plastic deformation; changes in physical and mechanical properties; and factors, such as temperature, influencing these changes. An account of the flow of metal in the industrial processes such as rolling, wire-drawing, &c., the formation of Luder's Lines or 'flow-lines', have been excluded because they belong more to the sphere of the mathematician and engineer than to that of the metallurgist. On the other hand, a short account of the effect of heat on deformed metal appeared essential before discussing the many theories that have been put forward.

One of the most difficult problems was to decide how much prior knowledge of the many aspects of the subject of plastic deformation of metals the reader could be assumed to have. A full account of the ordinary metallurgical processes referred to, the different forms of mechanical and physical testing, crystallography, and the use of X-rays in elucidating the structure of metals (to mention only the most important) was obviously out of the question. This information can be obtained in text-books and in books devoted entirely to the subject concerned, and its omission, even in an abbreviated form, in the present case has therefore been intentional.

Even with such limitations, it has not been possible to do more than give an outline of the most important aspects of the problem, and the reader is referred to the literature for further information.

My aim has been to avoid unnecessary experimental details and to lay stress on methods, particularly those employed in investigations of the distortions of single crystals, as no connected account of this work has yet been given. I have attempted to differentiate between facts established by experiment, inferences to be drawn from them, and theory, and to present the subject with impartiality, so that the reader can draw his own conclusions.

Wherever possible, the first published account of a particular research has been given unless later ones definitely supersede it or very much add to the information. Where a large amount of research has been published on any one subject, the results have been summarized, paying due regard to those most likely to be correct. Discrimination is necessary in such cases, but important differences have been noted. New work is constantly appearing, superseding or modifying previous results, and it is only by sifting the evidence very carefully that one can hope to extract the essential facts that will withstand the test of time.

I wish to express my thanks to Dr. G. W. C. Kaye and Mr. W. A. Wood of the Physics Department at the National Physical Laboratory for preparing the X-ray photographs on Plate IV. Dr. H. Gough, of the National Physical Laboratory, very kindly supplied me with Fig. 6, Plate V, and gave me permission together with Messrs. Ernest Benn, Ltd. to reproduce Figs. 52, 57, and 58 from his book *The Fatigue of Metals*. I wish to thank Dr. Gough also for a helpful criticism of Chapter VIII. I am indebted to Prof. Dr. E. Schmid of Freiburg, Switzerland, for Figs. 1, 2, and 3, Plate III, and to Edward Arnold & Co., for permission to reproduce Fig. 54 from Professor W. E. Dalby's *Strength and Structure of Steel and other Metals*. My thanks are also due to the Councils of the following Societies and Institutions for permission to reproduce diagrams and photographs from their publications: The Royal Society, The Institute of Metals, The Iron and Steel Institute, The Institution of Mechanical Engineers, and the Faraday Society.

Finally, my thanks are due to Dr. W. Hume-Rothery for reading the proofs and for his valuable criticism.

C. F. T.

ENGINEERING LABORATORY
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August 1934

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 FIG. 3. Extended copper-aluminium crystal.
 FIG. 4. Extended brass crystal. C. F. ELAM, *Proc. Roy. Soc. A*, 115 (1927), 148.

PLATE II.

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- FIGS. 1 and 2. Compression disks of aluminium crystals. G. I. TAYLOR and W. S. FARREN, *Proc. Roy. Soc. A*, 111 (1926), 529.
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PLATE III.

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- FIGS. 1, 2, and 3. Photographs of twin formation in zinc, bismuth, and cadmium. E. SCHMID.
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PLATE IV.

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- FIG. 1. Laue photograph through aluminium consisting of a number of large crystals.
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 FIG. 3. Debye rings in worked magnesium.
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 FIG. 5. Reflection photograph of rolled copper showing 'preferred' orientation.
 FIG. 6. The same as Fig. 5, illustrating structure obtained when crystals of considerable size are orientated as in rolled state.
 FIG. 7. (211) line in tungsten steel. (a) Separation of K_{α} doublet, no distortion; (b) line diffuse, distortion.

PLATE V.

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- FIGS. 1, 2, and 3. Cast and rolled aluminium. H. C. H. CARPENTER, C. F. ELAM, *J. Inst. Metals*, 25 (1921), 259.
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I

INTRODUCTION

METALS are usually composed of aggregates of small crystals, each of which is seldom bounded by regular plane faces such as are associated with crystals deposited from a solution. They normally have irregular boundaries which are produced where a number of grains growing from different centres ultimately meet. This is the normal procedure when metals solidify from the liquid state, and the size of the individual crystal depends chiefly on the rate at which the metal has been cooled through the melting-point. With the exception of castings, most metals for commercial purposes are worked and heat-treated in order to form them into suitable shapes, and in order to give them certain physical and mechanical properties. In these the crystal size is sometimes extremely small, fineness of grain being frequently very desirable. But in spite of their very small size and the absence of regular external form, each grain possesses all the properties of a crystalline substance. A crystal may be defined as a solid in which the atoms or molecules of which it is composed are arranged in a regular manner according to a definite geometrical pattern. The properties in different directions in a crystal are consequently not always the same, the differences being greater or less according to the symmetry of the crystal. In dealing with the properties of metals, therefore, it is necessary to take into consideration the properties of the individual crystals, which are anisotropic, as well as those of the aggregate, which are in the main isotropic. These differences are very pronounced in the behaviour of metal crystals and aggregates towards deformation.

The readiness with which metals will alter their shape and take up a desired form, i.e. their ductility, constitutes one of their most important properties. The actual process of change of shape may be considered to take place in one of two ways: (1) the metal may behave as a viscous fluid; (2) it may behave as an aggregate of grains in which each one deforms according to laws fixed by the nature of its structure, the sum of their deformations producing the required change of shape of the whole. Since it is now known that metals are crystalline aggregates, the second hypothesis suggests itself as being the more probable. It is also supported by experimental evidence.

The first effect of deformation on all solid substances is elastic in nature—i.e. it is not permanent; as soon as the force tending to produce this deformation is removed, any distortion that has taken place is also removed. If still greater forces are employed one of two things may happen: (1) the substance may deform plastically i.e. it will not return to its original shape when the force is removed; (2) it may fracture without deformation. Substances belonging to Class 1 are termed ductile: those belonging to Class 2 are termed brittle. In practice we have both types and variations and combinations of both types, in many cases depending upon the conditions under which the tests are carried out. Moreover, it is well known that even the most ductile substances will ultimately fracture. We are here concerned only with the changes accompanying plastic deformation of metals, and these will include the change of shape; the forces required to bring it about; the effects of varying conditions, such as temperature, rate of deformation, etc.; the change of properties accompanying the change of form; and finally fracture.

Ewing and Rosenhain† were the first to observe the sudden appearance of a series of parallel lines on the polished surface of metal crystals when a polycrystalline specimen was deformed. Similar bands were already known to occur on such minerals as rock salt, galena, etc., and had been previously described by Mügge‡. Ewing and Rosenhain were of the opinion that these lines represented the traces of planes of slip on the surface of the crystals, and that the deformation was caused by the gliding of parts of the crystal in layers along these planes. The markings on the polished surface were consequently called slip-bands. The definite correlation of slip-bands with internal structure and the actual process of deformation could only be determined when isolated metal crystals were available of such a size as to make possible external measurements of the change of shape, and X-ray determinations of the crystal orientation.

Before giving an account of these researches it is necessary to give a short summary of some of the methods employed in making isolated metal crystals.

The methods employed by various investigators fall into one of three classes, i.e. they can be made:

1. from the molten metal.

† A. Ewing, W. Rosenhain, *Phil. Trans. A*, **193** (1899), 353; **195** (1900), 279.

‡ O. Mügge, *Neues Jahrb. Mineral.* **1** (1886), 183; **1** (1898), 71; **2** (1899), 55.

2. by straining and heat-treating a fine grained metal in such a manner as to promote crystal growth in the solid state.

3. by deposition from the vapour phase or by decomposition of a volatile compound.

1. Growth of crystals from molten metal

The size of the crystals in a casting depends chiefly on the rate with which the metal passes through the freezing range. The crystals may be very large in a large casting, but usually the shape is not such that test-pieces of a reasonable size can be cut from them.

Obreimow and Schubnikow† and Bridgman‡ melted metals in a graphite or hard glass tube and, instead of allowing the metal to solidify normally, lowered the tube through a furnace so that it must cool from one end only. If the end of the tube is tapered, one crystal nucleus tends to form instead of more. Crystals twelve inches long and up to one inch diameter have been made by this method or variations of it, and it has been applied to a number of metals.

The orientation of the crystals produced by this method is generally haphazard, but Bridgman has reported that with non-cubic crystals the principal cleavage plane tends to lie parallel to the axis of the tube. In order to obtain a number of test-pieces of variable orientation, he§ replaces the straight tube by a network of glass tubes joined together at different angles. He finds that the crystal maintains its original orientation throughout, so that along different arms of the glass tubing, i.e. at each bend, a different crystal axis lies in the axis of the little rod. This method has been applied to alloys as well as to pure metals.

Another method in which the crystal is grown from the molten metal is that of Czochralski|| and Gomperz.†† A silica rod with a small crystal attached to act as a nucleus, just dips into a crucible containing the molten metal. It is slowly withdrawn, and a jet of carbon dioxide or nitrogen playing upon it, just at the point of contact, induces progressive solidification. If the temperature control and rate of withdrawal are correct, a wire, about 2 mm. in diameter, consisting of one crystal is obtained. This method is only applicable

† Obreimow, Schubnikow, *Z. Physik*, **25** (1924), 31.

‡ P. W. Bridgman, *Proc. Amer. Acad. Sci.* **58** (1923), 165; **60** (1925), 305.

§ *Ibid.* **63** (1929), 351.

|| J. Czochralski, *Z. phys. Chem.* **92** (1918), 219.

†† E. V. Gomperz, *Z. Physik*, **8** (1922), 184.

to metals of low melting-point, and has been used for the preparation of crystals of tin, zinc, cadmium, bismuth, and alloys of cadmium-zinc, tin-zinc, etc.†

Kapitza,‡ in his experiments on the behaviour of bismuth crystals in a magnetic field, wished to grow crystals with a particular orientation. His method was to weld a bismuth crystal on to a small rod of the metal, 1 to 2 mm. in diameter, at such an angle that the rod, if converted into a crystal, would have its principal cleavage plane parallel to the cleavage plane of the piece of crystal which was to act as nucleus. This was then placed on a copper plate and electrically heated from one end. The rod melted but, owing to surface tension and an oxide film, did not lose its shape. As soon as it had melted up to the point where it was joined to the piece of crystal, the plate was cooled. As the metal solidified, it followed the orientation of the piece of crystal. Rods 7 to 10 cm. long were made in this way.

2. Growth of crystals in solid state

The most important method depends on the fact that if a polycrystalline metal is strained and subsequently heated, recrystallization will take place and certain crystals will grow at the expense of others. This phenomenon is described in Chapter XIV. It is sufficient here to point out that it has been found that the size of the crystals so produced depends on the degree of deformation and the temperature of annealing. The smallest amount of strain that will induce recrystallization at all, produces the largest crystals. The general procedure is as follows:§

(1) Carefully anneal the rod or strip in which it is required to grow the crystal, so that grains are uniform in size and free from strain. The grain size is specially important in the case of iron,|| but not so important in aluminium.

(2) Strain the test-piece. This is generally done by pulling in a tensile testing machine until the specimen has extended 2–3 per cent.

(3) Heat for a further period. The temperature can only be found by trial and error in each case. Speaking generally, there will be no

† P. Rosbaud, E. Schmid, *Z. Physik*, **32** (1925), 197.

‡ P. Kapitza, *Proc. Roy. Soc. A*, **119** (1928), 358.

§ H. C. H. Carpenter, C. F. Elam, *Proc. Roy. Soc. A*, **100** (1921), 329.

|| C. A. Edwards, L. B. Pfeil, *J. Iron Steel Inst.* **109** (1924), 129.

growth below a certain temperature, but the lowest temperature at which growth will take place is the best, because there is less likelihood of growth starting from many centres. Too high an annealing temperature invariably produces smaller crystals. The best method is to start annealing below the temperature at which it is known from previous experiments that growth will begin, and then to raise the temperature slowly, finally finishing with as high a temperature as possible to complete the absorption of small crystals that tend to remain on the surface. The orientation of the crystals is sometimes influenced by the direction of the applied stress.

Iron crystals can also be made by this method.† There are certain special difficulties here. Crystal growth is very much impeded by the presence of carbide, so that it is essential to remove it by prolonged heating in a stream of moist hydrogen. The preliminary grain size is also important, and the final annealing temperature is limited by the allotropic change which takes place in pure iron at 900° C.

The method has been used to make crystals of aluminium-zinc alloys and β brass, but fails for those metals which twin readily after straining. Large crystals develop but they are always intersected by twin bands.

There are other methods depending essentially on working followed by heat treatment, but in these cases the straining is of a severe nature.

The tungsten filaments in electric bulb lamps are subjected to a very high temperature following severe working, and the crystals frequently become very large and occupy the whole cross-section of the wire.‡

Compressed tungsten powder will,§ under certain conditions, on sintering unite to form bars consisting of one crystal.

The Pintsch|| method for making tungsten wire depends upon pressing the fine grained powder through dies, at such a temperature and at such a rate that the issuing wire consists of one long crystal.

In almost all metals which are subjected to prolonged annealing after working, the crystals will grow to a greater or less extent, and these can sometimes be cut out for experimental purposes.

† Ibid.

‡ F. S. Goucher, *Phil. Mag.* (6) **48** (1924), 229.

§ Z. Jeffries, *J. Inst. Metals*, **20** (1918), 109. H. Alterthum, *Z. phys. Chem.* **110** (1924), 1-16.

|| W. Büttger, *Z. Elektrochem.* **23** (1917), 121.

3. The deposition of crystals from the vapour phase

Zinc is the most volatile of the commoner metals and can be distilled. Very beautiful allotriomorphic crystals of considerable size are deposited. The action of zinc vapour on β brass crystals has been used to grow γ brass crystals.[†] These form by a process of diffusion, and are crystallographically related to the β brass crystal.

Van Arkel, J. H. de Boer and J. D. Fast,[‡] and F. Koref,[§] have grown crystals by decomposing a volatile compound of the metal in such a way that the metal is deposited on a filament already consisting of one crystal. By this means they have grown large tungsten crystals, and also prepared samples of the metals zirconium and hafnium. The compounds used are tungsten tri-iodide and hexachloride, etc.

It is very rarely that the crystals produced by any of these methods have any fixed orientation relative to the shape of the test-piece. Neither are the methods such as to produce regular crystal faces which may enable the orientation to be determined. It is essential for all experiments on plastic deformation that the orientation of the crystal relative to the specimen be known with accuracy. This may be achieved by optical methods or by means of X-rays.

4. Determination of orientation

1. *Optical methods.* The metal must first be etched to develop the crystal facets. The facets are due to preferential solution along crystallographic directions. As an etched crystal is rotated in a beam of light, the surface appears light or dark according as the facets reflect or not.||

An ingenious method of plotting these reflections is due to Bridgman.^{††} The crystal in cylindrical form is etched and put through a hole in a wooden sphere. A distant source of light falls on to the crystal and a small piece of mirror which can be moved about in contact with the sphere. When a good reflection is obtained from the crystal the mirror is adjusted so that it also reflects. If the back of the mirror is blacked, a mark is made on the sphere at this point and represents the approximate position of the pole of a plane corresponding to the crystal plane which reflects by means of the etch facets.

[†] C. F. Elam, *J. Inst. Metals*, **53** (1930), 217.

[‡] A. E. van Arkel, J. H. de Boer, *Z. anorg. Chem.* **148** (1925), 345.

[§] F. Koref, *Z. Elektrochem.* **28** (1922), 511.

|| See Tammann, *Handbuch der Metallkunde*, 4th edition.

^{††} *Proc. Amer. Acad. Sci.* **60** (1925), 306.

Other methods of measuring the angles of these facets have been devised and may be found described in the literature dealing with this subject.†

2. The most accurate methods are those involving the use of X-rays. It is only proposed here to refer to these very briefly.‡

(i) *Laue methods*. A single Laue photograph is usually sufficient to determine the orientation completely. It is not possible, however, to measure large crystals in this way, owing to the absorption of the X-rays, and although it is possible for Laue spots to be reflected from the surface like reflections with monochromatic rays, the exposure necessary is generally long.§

(ii) *With monochromatic rays*. All the usual methods of crystal analysis are available, although these have to be specially adapted when large crystals are to be examined. The difficulties are rather in the direction of holding and mounting the specimen in the X-ray beam than in taking or interpreting the photographs.||

The essential aim in all methods is to fix the orientation of the crystal to some arbitrary plane in the specimen, which can likewise be made the reference plane for all distortion measurements. How this is done will be described later in connexion with the distortion measurements.

† J. E. Stead, *J. Iron Steel Inst.* **53** (1898), 145; **54** (1898), 137. E. Heyn, *Mitt. K. Techn. Vers.-Anst.* **16** (1898), 310. J. Czochralski, *Z. anorg. Chem.* **144** (1925), 131. G. Tammann, *Z. anorg. Chem.* **148** (1925), 293. P. Shimizu, *Sci. Rep. Tohoku Univ.* (1) **16** (1927), 621. J. Weerts, *Z. techn. Phys.* **9** (1928), 126. G. Sachs, *Z. Metallkunde*, **17** (1925), 299.

‡ For a short account of X-ray crystal analysis, see R. W. James, *X-ray Crystallography*, Methuen's Monographs of Physical Subjects. A full account is to be found in *The Crystalline State*, by W. H. and W. L. Bragg. G. Bell & Sons.

§ For particulars as to methods of interpreting Laue photographs and of determining the orientation, the reader is referred to the literature dealing with X-ray crystal structure, and to the following papers: R. Gross, *Centralbl. Min.* (1920), 52. E. Schiebold, G. Sachs, *Z. Krist.* **63** (1926), 34. v. Göler, G. Sachs, *Naturwiss.* **16** (1928), 412.

|| Here again reference should be made to the literature: A. Müller, *Proc. Roy. Soc. A.* **105** (1924), 500. K. Weissenberg, *Z. Physik*, **23** (1924), 229. H. Mark, M. Polanyi, E. Schmid, *Z. Physik*, **12** (1922), 58.

II

DISTORTION OF METAL CRYSTALS†

A CRYSTAL is essentially an anisotropic solid. The atoms are arranged in planes and lines with different spacing between them causing differences in cohesion along these directions and planes. An isotropic body will not materially change its shape when it is pulled in tension. For example, a round test-piece may elongate, but still remain circular in cross-section. Crystals present a totally different problem. They can, and do, only deform in certain specified ways, leading to a complete change of shape even when they are subjected to normal tension or compression. A round bar will become elliptical in cross-section; a rectangular bar, rhombic. Photographs of metal crystal test-pieces are shown in Plate I. These have been chosen out of a large number as representative types. The brass test-piece (Fig. 4, Plate 1) was originally marked according to the scheme illustrated in Fig. 4. It was in an endeavour to explain the reason for these changes that the analysis of the deformation of single crystal metal test-pieces was undertaken. Similar measurements were carried out almost simultaneously on aluminium and zinc crystals, and form the basis for all subsequent work on the deformation of metal crystals. They will be considered somewhat in detail.

In the following pages care has been taken to use a uniform notation in referring to crystal planes and axes, and as far as possible to conform to the conventional terms of crystallography. This is essential in order to avoid confusion, and failure to keep to some recognized form has made the subject in the past more difficult to follow than need have been the case.

Crystallographers have fixed certain directions, called the crystal axes, to which to refer all planes and directions in the crystal. These are chosen with regard to the symmetry of the crystal, and are generally three lines parallel to the intersection of three important crystal faces which do not lie in the same plane. For example, in a cubic crystal three edges of the cube, giving rectangular axes, are chosen.

The lengths of the axes and their angular relationships are fixed by

† The elements of crystallography are contained in references in Chap. I, see footnote, p. 7.

the symmetry of the crystal, and if the crystal axes are referred to as rotating in relation to a plane or direction in a test-piece it means that the whole crystal is so rotated. In addition to the 'reference' axes a crystal has axes of symmetry, and it has also become customary to refer to other directions in the crystal as axes. It is important to distinguish between *directions* in a crystal and *planes*. In the following pages, round brackets, (), are used for a particular crystal plane to which the appropriate indices and signs are given. If the indices are enclosed in curly brackets, { }, all or any of the planes belonging to the form are indicated. Square brackets, [], are used to specify directions.

1. The distortion of zinc crystals

Mark, Polanyi, and Schmid† worked with zinc crystals in the form of wire 2 mm. diam. made by Czochralski's and Gomperz's method (p. 3). Some crystals were always brittle; others could be extended to four and five times their original length, and at 205° C., seventeen times. The crystals became elliptical in cross-section and finally almost flat. Well marked slip-bands appeared on the surface.‡ At the temperature of liquid air fracture was parallel to the glide-planes, but other planes were involved at 180°–200° C.

Two independent sets of measurements were made: (1) the inclination of the glide-planes and other markings on the surface, plane of fracture, etc., to the axis of the wire (these measurements were carried out with the aid of a microscope): (2) X-ray determination of crystal axes.

An examination of the crystal band after extension showed that the axis of the ellipses made by the glide-planes did not coincide with the axis of the ellipse of the band itself. The following data were required:

- (1) the axes of the glide-plane ellipse.
- (2) the angle between the glide-plane ellipse and the axis of the wire.
- (3) the angle between the major axis of the ellipse and a plane through the centre of the band, in order to measure the eccentricity of the ellipses.

† H. Mark, M. Polanyi, E. Schmid, *Z. Physik*, **12** (1927), 58.

‡ Fig. 2, Plate I, which is a photograph of a copper-aluminium alloy crystal, very much resembles a distorted zinc crystal.

Fig. 1 explains the measurements and the geometrical determinations made from them. a , b , and Δ were measured by means of a microscope with micrometer eye-piece. From these measurements the angles μ' and ν are obtained direct. d and h are the thickness and breadth of the band.

$$OA = \sqrt{[\frac{1}{2}(a+b) - \Delta]^2 + \frac{1}{4}d^2}, \quad OB = \frac{h}{2 \sin \mu'}$$

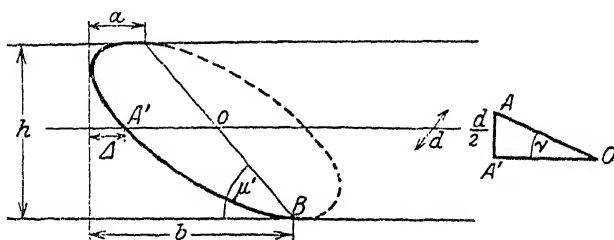


FIG. 1.

The angle between the axes μ being obtained, $\cos \mu = \cos \mu' \cos \nu$, the lengths of the axes required are

$$A = \frac{1}{2} \{ \sqrt{(OA^2 + OB^2 + 2OA \cdot OB \sin \mu)} + \sqrt{(OA^2 + OB^2 - 2OA \cdot OB \sin \mu)} \},$$

$$B = \frac{1}{2} \{ \sqrt{(OA^2 + OB^2 + 2OA \cdot OB \sin \mu)} - \sqrt{(OA^2 + OB^2 - 2OA \cdot OB \sin \mu)} \}.$$

The angle χ which the ellipse plane makes with the axis of the wire is obtained thus:

$$\chi = \sqrt{(\cos^2 \rho_0 + \cos^2 \sigma)},$$

where $\tan \rho_0 = \frac{\tan \nu}{\sin \mu'}$ and $\tan \sigma = \frac{\tan \mu'}{\sin \nu}$.

The angle β which the major axis makes with the cross-section OA is derived from the formula

$$\tan \beta = \frac{\tan \mu}{2} \left(1 - \frac{B^2}{A^2} \right) \pm \sqrt{\left(\frac{\tan^2 \mu}{4} \left(1 - \frac{B^2}{A^2} \right)^2 - \frac{B^2}{A^2} \right)},$$

in which the sign is plus if $OA < OB$ and minus for $OA > OB$.

The angle between the middle plane of the crystal band and the normal to the glide ellipse is

$$\tan i = \sin \nu \cot \mu.$$

Similar measurements and calculations were carried out with any other visible markings on the surface and with the plane of fracture.

X-ray analysis of the crystal was made by the rotation method.

The crystal lattice was unaltered by the deformation, but the reflections from all the planes became less well defined. The measurements required were:

(1) The angle (δ) between the hexagonal axis (zinc crystallizes in a hexagonal close-packed lattice) and the wire axis. The basal plane is perpendicular to the hexagonal axis.

(2) The angle (π) which the projection of the wire axis on to the basal plane of the zinc lattice makes with the nearest digonal axis of the first order. The basal plane is the plane in which the atoms are most closely packed. The digonal axes, of which there are three at 60° to each other, are formed by the intersection of the first order prismatic planes with the basal plane. They represent lines of atoms in closest formation on the basal plane.

(3) The angle γ between the hexagonal axis and the plane of the crystal band.

The two sets of measurements, i.e. those concerned with the glide planes, etc. and those of the crystal orientation were correlated by geometrical calculations,[†] and it was shown by means of a model, that the change of shape observed could be obtained by means of the gliding of a number of planes over one another in a particular direction. If l_0 and l_1 are the lengths of a crystal before and after extension and δ_0 and δ_1 the angles between hexagonal axis and wire axis before and after extension respectively, it can be shown that

$$\frac{l_1}{l_0} = \frac{\sin \delta_0}{\sin \delta_1}.$$

Actual measurements and calculations before and after extension agreed very closely. Further, the glide ellipses were found to coincide with the basal plane of the hexagonal lattice. Fig. 2 illustrates the method of deformation diagrammatically.

The eccentricity of the glide-plane ellipse, with respect to the section of the band, shows that the direction of slip did not coincide with the direction of maximum shear stress, i.e. the line of greatest slope in the basal plane. The fact that the crystal remains a crystal after extension is evidence that slip follows a definite crystallographic direction. The most important of these in the basal plane are the three digonal axes. It was therefore probable that the direction of slip was along

[†] It has not been thought necessary to give details of these calculations here. They are given fully in the original paper.

the digonal axis nearest the direction of maximum stress. Since the direction of slip in the basal plane remains constant throughout the extension, the angle which this line makes with the wire axis, λ ,

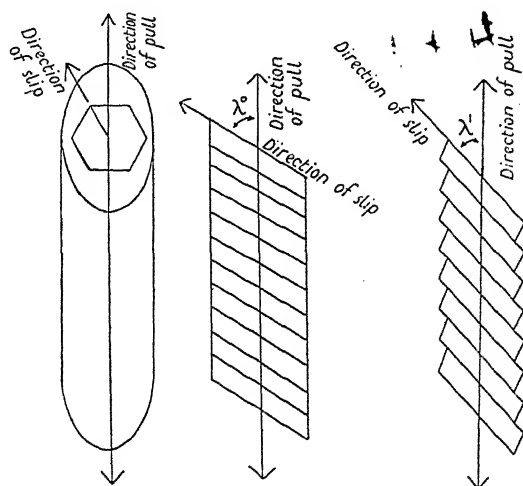


FIG. 2. Distortion of zinc crystal.

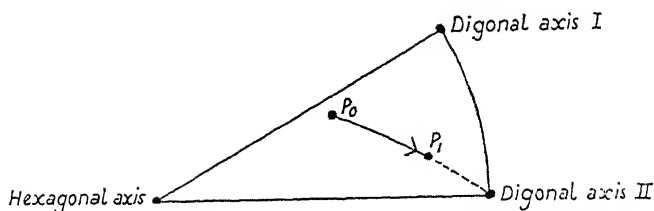


FIG. 3. Diagram illustrating movement of axis of zinc crystal wire during extension.

alters as the inclination of the plane alters; therefore, the extension
 $l_1 \frac{\sin \lambda_0}{\sin \lambda_1}$ (Cf. above.) The normal to the slip-plane moves relative to the wire axis in a plane containing its original position, the direction of slip and the wire axis. According to evidence derived from the model and geometrical considerations, the direction of slip is obtained from the intersection of that plane towards which the wire axis moves during extension with the basal plane. This is illustrated in projection in Fig. 3 and the movement of the axis towards the digonal axis is clearly seen.

The authors showed, therefore, that the process of extension of the

zinc crystal could be entirely accounted for by means of slip on the basal (0001) plane, and in a direction of that diagonal axis [1010], which coincided most nearly with the direction of maximum shear stress.

In addition to the glide planes, the plane of fracture was also measured. At low temperatures it was parallel to the basal planes, but at higher temperatures planes closely approaching 90° to this were obtained and were thought to be prism faces. The formation of these planes and a sudden drawing down of the crystal just before fracture (*Nachdehnung*) have subsequently been shown to be connected with twin formation and will be referred to later.

This investigation provided an explanation for the apparent division of crystals into ductile and brittle. For it can be seen that if the basal plane is already, in the undistorted crystal, in such a position that the angle between it and the axis is small, very little extension can take place. Similarly, if the plane is nearly perpendicular to the axis, slipping becomes very difficult. The end position of the basal plane was found to vary with the temperature of the test. At 20°C . it was $10\text{--}20^\circ$; at $120\text{--}180^\circ\text{C}$. it was $8\text{--}10^\circ$; and at 200°C . it was $2\text{--}4^\circ$.

2. The distortion of aluminium crystals

The method employed by Mark, Polanyi, and Schmid is inapplicable to metals where the slip-bands and cleavage planes are not clearly marked, and is extremely difficult where slip takes place on more than one plane. A geometrical analysis of the change of shape of a metal crystal test-piece, such as was first used for aluminium,[†] does not depend on surface markings and can be applied to all types of deformation. The only requirement is that the crystals shall be sufficiently large to enable suitable measurements to be made.

Large crystals of aluminium made by the method of straining and heat-treating,[‡] could be obtained of 1.25 cm. diam. and up to 25 cm. in length. Some of these were tested round; others were machined square.

Round bars became elliptical in cross-section on extension and slip-lines appeared on the surface (Fig. 1, Plate I). They were very fine and it was found that single lines could not be traced round a bar.

[†] G. I. Taylor, C. F. Elam, *Proc. Roy. Soc. A*, **102** (1923), 643.

[‡] H. C. H. Carpenter, C. F. Elam, *Proc. Roy. Soc. A*, **100** (1921), 329.

It seemed probable, however, that the deformation proceeded by means of slip on certain crystallographic planes and in certain crystallographic directions, as X-ray analysis showed the symmetry of the crystal to be unaltered by the test, and the angles between reflecting planes only slightly altered.

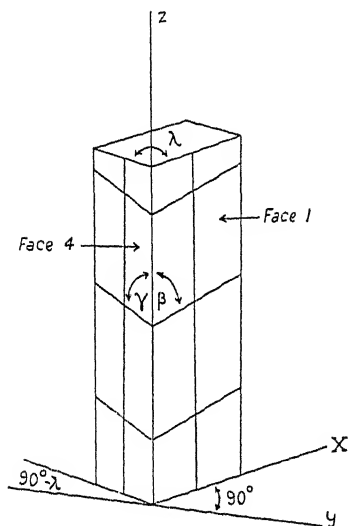


FIG. 4. Method of marking rectangular test-piece for distortion measurements.

When a sphere is transformed by a uniform strain into an ellipsoid, the directions which are unchanged in length lie on a quadric cone. The cone passes through the curved line of intersection of the strain ellipsoid and the sphere, when the two are placed concentrically. The particles in this cone have two positions corresponding to the strained and unstrained material. Applying this to the crystal, the problem was to determine a number of directions in the crystal which did not change in length during the deformation.

This was done in the following way.† As the method of measuring and calculating the change of shape is somewhat simpler in the square bars than in the round, this only will be described. All four faces were marked with fine scratches both vertically and horizontally. Fig. 4, Plate I, shows a rectangular crystal of brass after extension. Measurements were made on all faces before and after pulling. Those required for the calculations were as follows:

- λ the angle between the faces 1 and 4.
- b the width of faces 1 and 3.
- c the width of faces 2 and 4.
- β the angle between the horizontal and vertical scratches on face 1.
- γ the angle between horizontal and vertical scratches on face 4.
- d the distance parallel to the axis of the specimen between successive horizontal scratches.

† G. I. Taylor, C. F. Elam, *Proc. Roy. Soc. A*, **112** (1926), 337. The method used here is not the same as that described in the earlier paper, *Proc. Roy. Soc. A*, **102** (1923).

Rectangular coordinates were chosen so that the edge formed by the intersection of faces 1 and 4 was the axis OZ and face 1 was the plane $y = 0$ (Fig. 4).

Using suffixes 0 and 1 to denote the conditions before and after stretching, the ratio of the final to the initial length is $\frac{d_1}{d_0} = \epsilon$. Also

$$\text{let } f = \frac{v_1}{b_0} \text{ and } g = \frac{v_1}{c_0}.$$

If (x_1, y_1, z_1) are the coordinates in the strained material of a particle whose coordinates in the unstrained material were (x_0, y_0, z_0) , the formulae of transformation are

$$\left. \begin{aligned} x_1 &= fx_0 - ly_0 \\ y_1 &= my_0 \\ z_1 &= px_0 + qy_0 + \epsilon z_0 \end{aligned} \right\} \quad (1)$$

where

$$\left. \begin{aligned} l &= g \frac{\cos \lambda_1}{\sin \lambda_0} - f \cot \lambda_0 \\ m &= g \frac{\sin \lambda_1}{\sin \lambda_0} \\ p &= -\epsilon \cot \beta_0 + f \cot \beta_1 \\ q &= \epsilon \cot \beta_0 \cot \lambda_0 - f \cot \beta_1 \cot \lambda_0 + g \frac{\cot \gamma_1}{\sin \lambda_0} - \epsilon \frac{\cot \gamma_0}{\sin \lambda_0} \end{aligned} \right\} \quad (2)$$

The unstretched cone is given by

$$x_0^2 + y_0^2 + z_0^2 = x_1^2 + y_1^2 + z_1^2, \quad (3)$$

and eliminating x_1, y_1, z_1 from (1) and (3) the equation of the unstretched cone becomes

$$x^2(f^2 + p^2 - 1) + y^2(l^2 + m^2 + q^2 - 1) + z^2(\epsilon^2 - 1) + 2xy(fl + p) + 2xz(\epsilon p) + 2yz(\epsilon q) = 0. \quad (4)$$

If spherical polar coordinates are used, θ being the angle which the direction makes with the axis of z and ϕ the angle which its projection on the plane $z = 0$ makes with the axis of x , then

$$\left. \begin{aligned} x/z &= \tan \theta \cos \phi \\ y/z &= \tan \theta \sin \phi \end{aligned} \right\} \quad (5)$$

so that the equation of the unstretched cone in its first position before stretching the material is

$$\{(f^2 + p^2 - 1)\cos^2 \phi + 2(fl + pq)\cos \phi \sin \phi + (l^2 + m^2 + q^2 - 1)\sin^2 \phi\}\tan^2 \theta + (2\epsilon p \cos \phi + 2\epsilon q \sin \phi)\tan \theta - (\epsilon^2 - 1) = 0. \quad (6)$$

To find the unstretched cone in its second position, in the stretched material, the simplest method is to reverse all the formulae, replacing measurements made before extension by corresponding ones in the stretched material and vice versa. The formulae of transformation are then

$$x_0 = f_1 x_1 + l_1 y_1,$$

$$y_0 = m_1 y_1,$$

$$z_0 = p_1 x_1 + q_1 y_1 + \epsilon_1 z_1,$$

where

$$l_1 = \frac{1}{g} \frac{\cos \lambda_0}{\sin \lambda_1} - \frac{1}{f} \cot \lambda_1, \text{ etc.}$$

and the equation to the unstretched cone in its second position is identical with (6) except that each of the symbols inside the brackets has a suffix 1.

Values for θ are now calculated using (6) and substituting different values for ϕ . Those chosen were 90° , 60° , 30° , 0° , -30° , -60° .

Having obtained the spherical polar coordinates of a number of these unstretched directions, a stereographic projection was made. This method is largely used by crystallographers and has recently been applied to depict all types of deformation in metal crystals. Consider a crystal surrounded by a sphere. The normals drawn to all the planes cut the sphere at points which are the poles of the corresponding planes and the angles between them, measured on the sphere, are the angles between the plane normals. If such a figure is viewed from one of the poles of the sphere and a line is drawn from the eye to a crystal pole, it will cut the plane of the equator at a point which is the projection of the crystal pole in this plane. By this means all the points on a sphere can be plotted on a single plane.

A stereographic net very much facilitates the plotting of points and measuring of angles.

It consists of a number of meridian circles and parallels of latitude projected on to the equatorial plane. In order to use it, a circle is drawn on a piece of tracing paper the size of the circle of the net, and a reference line, in this case representing Face 1, corresponding to the equator. If θ and ϕ are the spherical polar coordinates of a point, and the centre of the circle represents $\theta = 0$ and the reference line $\phi = 0$, then ϕ is measured off from the reference line on the circumference, counting the degrees on the net, and a second line drawn joining this point to the centre. Now rotate the tracing paper until this second line coincides with one of the diameters of the net and

mark the value of θ , counting the degrees from the centre. This is the required point. It is a convention to mark all points lying on the upper hemisphere with a dot and all those underneath with a cross. To measure the angle between two points, turn the tracing paper until the two points lie on the same great circle and count the degrees between them on the great circle of the net. All the 'unextended'

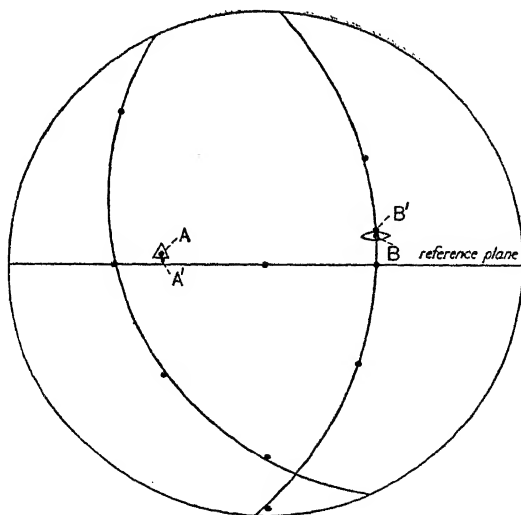


FIG. 5. Diagram showing unextended cone consisting of two planes. A' is position of normal to slip-plane as measured by distortion; A is position of normal to slip-plane as predicted by X-rays; B' is direction of slip as measured by distortion; B is direction of slip as predicted by X-rays.

Only the points on the upper half of the sphere are shown for each plane, so as to avoid confusion.

A = pole of (111) plane. B = pole of ($\bar{1}$ 01) plane.

directions can be plotted in this way, and also the poles of the crystal planes determined by means of X-rays.

When this is done, it is found that all the points lie on two great circles, i.e. the 'unextended' cone consists of two planes (Fig. 5). It is interesting to note, that the figure obtained from similar measurements made on a normal polycrystalline bar would be a circular cone, whose axis agreed with the centre of the figure. Comparing the positions of the principal crystallographic planes with the planes determined from distortion measurements, one of the latter was found to agree with one of the octahedral $\{111\}$ planes of the crystal (A in Fig. 5)

at all stages of the deformation (up to a point which will be dealt with later), whereas the other plane had no relation to the crystal structure and changed its position relatively during the extension. Thus the deformation can be considered as being due to shear on one of the octahedral planes of the crystal.

The direction of shear can also be obtained from the measurements. Let $ABCD$ be a rectangle in the plane perpendicular to the plane of slip and let AD be in the plane on which slipping takes place. Let the sheared position of BC be $B'C''$. All the particles in the line $BB'C''$

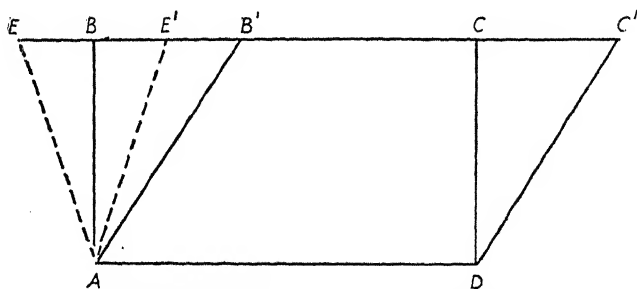


FIG. 6. Diagram illustrating method of determining direction of slip and amount of shear.

shift through a distance equal to BB' . One of the unextended lines of particles is evidently AD . The other is found by taking two points E and E' at distances equal to $\frac{1}{2}BB'$ on opposite sides of B . The line EA in the unstrained material moves to $E'A$ in the strained material. Since $EA = E'A$, the lines EA and $E'A$ are the traces of the second unextended plane. The direction of shear is in the line at right angles to the line of intersection of the two unstretched planes. When slipping is confined to one plane, the second undeformed plane lies at equal angles on opposite sides of the plane which is perpendicular to the plane of slip and the direction of slip, i.e. the line AB , in the strained and unstrained positions respectively. Hence, in order to determine the direction of slip from the stereographic figures of the two unstretched planes, it is only necessary to mark off the point in the slip-plane which is at 90° to the intersection of the two planes.

In all crystals investigated, this point coincided with the pole of a plane of indices $\{110\}$. There are three such planes which intersect each octahedral plane and their line of intersections are lines of atoms of closest arrangement in this plane. On general grounds, therefore, these are the most probable directions of slip.

During the process of deformation, the orientation of the crystal relative to the axis of the test-piece is changing continuously. This can be shown most clearly in the following way. If the figure in which the axis is in the centre is rotated by means of the stereographic net until the pole of a $\{100\}$ plane comes into the centre and all the other points moved to a corresponding amount, a symmetrical

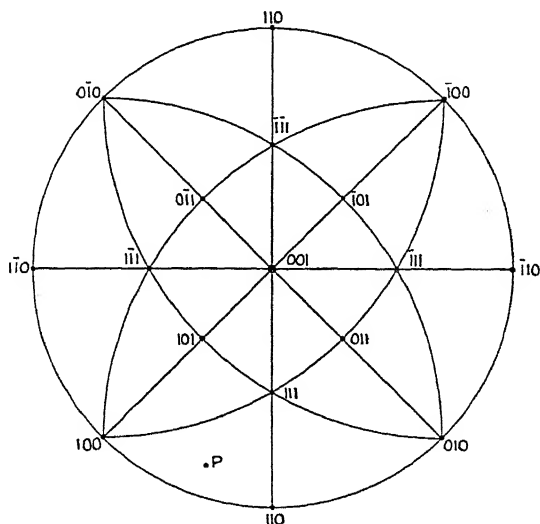


FIG. 7. Stereographic projection of cubic crystal.
 P = axis of specimen.

figure is obtained showing the position of all the principal cubic planes. The point representing the axis of the test-piece now falls in one of 48 crystallographically similar triangles formed by the normals to the $\{111\}$, $\{110\}$, and $\{100\}$ planes (Fig. 7).[†] If such a figure is made for different stages of the extension, the test-piece axis will move relative to the crystal axes along the great circle through the point of origin and the pole of that $\{110\}$ plane which is the direction of slip. Figs. 8–11 are part of the complete projection, consisting of two triangles, illustrating the movement described. They are characteristic of this type of distortion and of all crystals having a face-centred cubic lattice that have been investigated. Having established the relation between distortion measurements and rotation of the crystal axes, the determination of the latter by means of X-rays only has

† Only 24 triangles situated on one half of the sphere are shown in the figure.

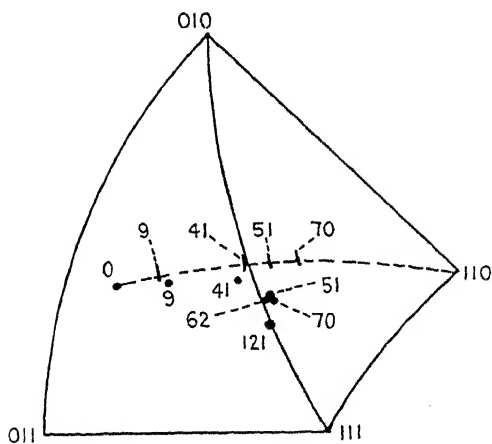
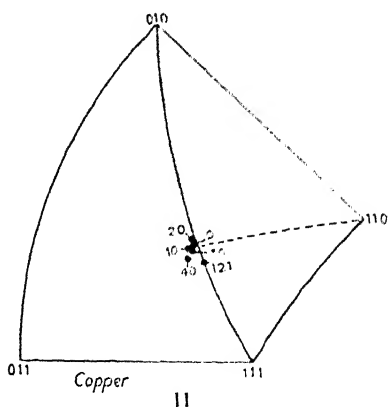
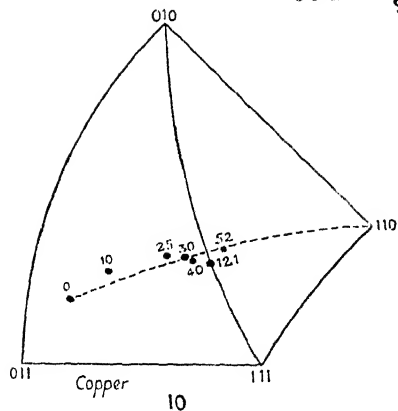
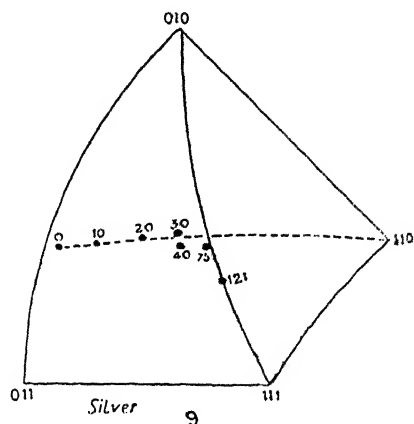


FIG. 8. Aluminium crystal. Movement of test-piece axis relative to crystal axes. Numbers on top refer to calculated position; numbers underneath refer to measured position for same extensions.



FIGS. 9, 10, and 11. Diagrams illustrating movement of axis of test-piece in relation to crystal axes. Numbers refer to measured extensions.

been used in many cases to investigate the deformation of other metals. Examples of this method are to be found in the work of Elam on crystals of copper, silver, and gold† and of Sachs‡ and his collaborators on aluminium, brass, etc.

During the later stages in the distortion of aluminium crystals, the 'unextended' cone no longer consisted of two planes (Fig. 12). It was concluded that possibly more than one crystal plane was con-

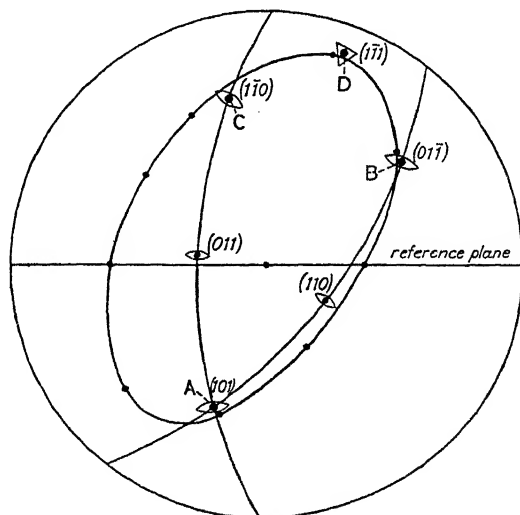


FIG. 12. Unstretched cone for extension during period of slip on two planes $(11\bar{1})$ and $(\bar{1}11)$.

tributing to the deformation. As the test-piece axis moves during the test towards the direction of slip, it ultimately reaches the boundary of its spherical triangle represented by the great circle through the poles of the (010) and (111) planes shown in Figs. 8–11. At any point along this line it is equally inclined to two $\{111\}$ planes, $(11\bar{1})$ and $(\bar{1}11)$, and both planes now begin to take part in the distortion. As the direction of slip is towards the pole of the (110) plane on the plane $(11\bar{1})$ and towards the pole of the (011) plane on the other $(\bar{1}11)$, the axis tends to move along this line until both directions of slip lie on the same great circle, i.e. to the pole of the (121) plane. No further movement then takes place. Crystals which are near this

† C. F. Elam, *Proc. Roy. Soc. A*, **112** (1926), 289.

‡ For references, see Table 1. No distortion measurements were made in any of these investigations.

point at the beginning of the deformation, slip simultaneously on both octahedral planes and the orientation relative to the test-piece axis remains almost constant throughout the test (Fig. 11). The calculated movement of the axis does not always agree exactly with that measured for the same amount of deformation. This may be attributed to errors in measurement, imperfections in the crystal and to the crystal not conforming exactly to rule, owing to the fact that it is itself broken up by the process of deformation.

It has been shown that the form of the unextended cone during the later stages of deformation is consistent with the theory of slip on two octahedral planes, but the asymmetry of the cone to the two planes, which is depicted in Fig. 12 is due to a greater amount of slip on the first plane. For proof that double slipping of the type suggested takes place in accordance with theoretical calculations, the reader is referred to the original papers.†‡

3. The distortion of crystals having a body-centred cubic lattice

The deformation of some metal crystals having a body-centred cubic lattice has been studied, e.g. tungsten, iron, and β brass.

Goucher§ broke tungsten wires consisting of crystals occupying the whole cross-section of the wire, and measured the angle of the wedge. He cut sections parallel to the slip-planes and took X-ray photographs to determine the orientation of the crystal. He concluded that the formation of the wedge was due to simultaneous slipping on two planes of type $\{112\}$ and in a direction $[111]$. From the nature of his experiments, only those crystals having an orientation such that two $\{112\}$ planes were equally inclined to the axis would deform in preference to others differently orientated.

The extension of the crystal depends on the cross-sectional area and the angle made by the glide-plane with the axis. The wedge angle was approximately 38° , but in a number of cases was larger. This could be accounted for if it was assumed that slip was not symmetrical on both planes.

Taylor and Elam|| carried out a series of measurements on iron

† G. I. Taylor, C. F. Elam, *Proc. Roy. Soc. A*, **108** (1925), p. 28.

‡ von Göler, G. Sachs, *Z. Physik*, **41** (1927), 103.

§ F. S. Goucher, *Phil. Mag.* **48** (1924), 800.

|| *Proc. Roy. Soc. A*, **112** (1926), 337.

crystals similar to those with aluminium. The 'unextended' cone consisted of two planes, but neither of them coincided exactly with any crystal plane. This can be seen from the figures, Fig. 13. The pole of one plane varied between those of $\{110\}$ and $\{112\}$ planes on the great circle perpendicular to a $[111]$ direction, which represented the direction of slip obtained from the distortion measurements. The position of the crystal axes relative to this plane and the $[111]$ direction, was almost constant, throughout the test. The main

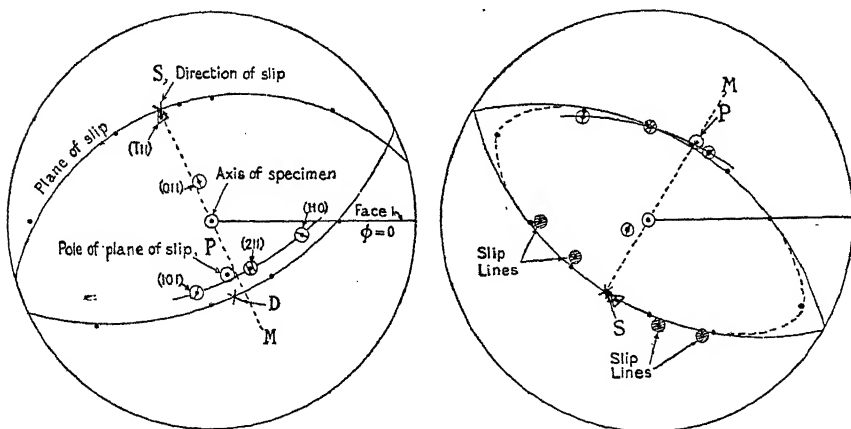


FIG. 13. $\epsilon = 1.0$. $\epsilon = 1.0915$. Distortion of iron crystal.

direction of slip-bands also agreed with the distortion plane. In the several crystals tested, the pole of the plane varied considerably. In one instance it almost coincided with the pole of the $\{112\}$ plane, while in another it was very near the pole of the $\{110\}$ plane. It was also noticed that the character of the slip-bands altered according to the angle which the faces of the specimen made with the slip-plane and direction of slip. If the traces of the slip-planes on a specimen face are nearly parallel with the direction of slip, the slip-bands are very regular and straight. If they are perpendicular to the direction of slip, they are broken up and confused, although they still maintain an average principal direction. The authors suggested that the deformation of iron could be compared with the deformation which would occur in a bundle of pencils, when these slid over each other in groups and layers. Looked at from the end, a section would show the traces of slip-planes to be ragged and uneven. A section parallel to the direction of slip, i.e. along the pencil axes, would show straight

slip bands. It is also equivalent to saying that slip occurs on more than one crystal plane in the same direction. The glide-plane determined from the distortion measurements depends chiefly on the stress distribution. This will be referred to again later.

Fahrenforst and Schmid† have suggested that a plane of type $\{123\}$ may be the slip-plane in iron. The poles of such planes lie on the same great circle as those of the $\{110\}$ and $\{112\}$ planes and in some cases very near to the poles of the distortion planes in the iron crystals. In other cases the disagreement is too great to be considered as error. They constructed diagrams representing the movement of the test-piece axis for slip on a $\{110\}$, $\{112\}$, and $\{123\}$ plane and for a plane of random orientation, all in a $[111]$ direction, and compared these with the results obtained from a number of iron crystals. They considered that the observed results were in closer agreement with slip on a $\{123\}$ plane than with any of the others.

Crystals of β brass‡ gave similar results to those of iron. No slip-bands were observed on the surface, but distortion experiments showed the direction of slip to be $[111]$ and the pole of the slip-plane to lie between planes of type $\{112\}$ and $\{110\}$ perpendicular to it. The tendency was for the plane to be near a $\{110\}$ plane and in some cases to coincide with it. von Göler and Sachs§ record an experiment on a single β brass crystal, made direct from the melt, in which slip-bands appeared in agreement with traces of a possible $\{110\}$ plane. In this case, however, the orientation was such that the plane of maximum shear stress was very close to the dodecahedral plane in question (see Chapter III).

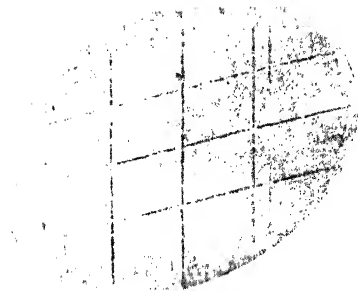
Some more recent work on these alloy crystals|| has confirmed the occurrence of slip-bands corresponding to $\{110\}$ planes in the crystal and suggests that the ultimate explanation of this form of distortion depends on the distribution of slipping on more than one plane of this type. For example, if there is an equal amount of slip on $(0\bar{1}1)$ and (101) (which are at 60° to each other in the direction $[\bar{1}11]$, which is common to both), the result would be the same as slip on the $(1\bar{1}2)$ plane which lies between them. As there are twelve planes of the type $\{110\}$, any of which, if suitably situated, may act as slip-planes, the choice is a wide one, and if two function simultaneously, an even

† W. Fahrenforst, E. Schmid, *Z. Physik*, **78** (1932), 383.

‡ G. I. Taylor, *Proc. Roy. Soc. A*, **118** (1928), 1.

§ von Göler, G. Sachs, *Naturwiss.* **6** (1928), 412.

|| C. F. Elam, *Nature*, **133** (1934), 723.



FIGS. 1 and 2. Compression disks of aluminium crystals



FIG. 4.
Side view of aluminium
test-pieces consisting of
two crystals



FIG. 3. Aluminium test-pieces consisting of two or
more crystals

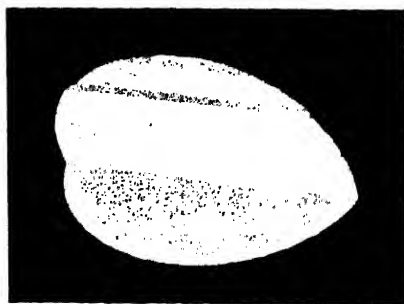


FIG. 5. Aluminium 'twin' produced by
deformation

greater variety of deformation may be expected. Some such solution of the problem appears more likely, than that planes of higher indices are the slip-planes.†

4. Compression tests

Similar results have been obtained with compression tests. Farren and Taylor‡ prepared small disks of aluminium which were suitably marked for distortion measurements and compressed between steel plates lubricated with grease. In this way a very uniform distortion was obtained. The round disk became elliptical in shape (Figs. 1 and 2, Plate II) and the distortion was found to be of the same nature as in tensile tests, i.e. slip on a $\{111\}$ plane and in a $[110]$ direction in the case of aluminium. The direction of slip in a compression specimen is exactly the reverse of what it would be if the same crystal were subjected to tension, if the normal to the faces is regarded as identical with the axis of the tensile specimen. When, however, the movement relative to this axis and the crystal axes is considered, the problem is somewhat different. In the tensile specimen, the traces of the slip-planes on the surface are constantly altering in angle with the axis. In compression specimens, the traces of the slip-bands on the surfaces are always perpendicular to the normals to the faces although the angle between the slip-planes and faces is continually changing. As all the crystal axes are fixed, relative to the lines made by the traces on the surfaces, the movement of the axis relative to the crystal axes is along the great circle passing through the normal to the disk and the normal to the slip-plane.

Here again, double slipping begins, when a symmetrical position of the crystal axes is reached, and experiment showed that the amount of slip on both planes was nearly equal.

It was also shown that compression tests on iron crystals were similar to tensile tests.

Further experiments on the deformation of β brass crystals by compression will be found in the section dealing with the forces required to produce deformation.

The methods just described have been used to determine the slip-plane and direction of slip of a number of pure metals and alloys. It is not necessary here to go into details of the experiments carried out

† See also Chapter VII for fatigue tests on iron crystals.

‡ G. I. Taylor, W. S. Farren, *Proc. Roy. Soc. A*, **111** (1926), 529. G. I. Taylor, *Proc. Roy. Soc. A*, **116** (1927), 16.

in each instance. A table of results has been made and references to the original papers given.

Metal or Alloy	Lattice	Slip-plane	Direction of Slip	Plane and Direction of greatest Atom Density		References
Aluminium	Face-centred Cubic	{111}	[110]	{111}	[110]	1-10
Copper						11, 18
Silver						11, 12
Gold						11
Copper-zinc α						13, 14, 15, 16
Copper-Aluminium α						17
Aluminium-Copper						18
Aluminium-zinc						19
Gold-silver						20
Copper-gold						21
Iron α	Body-centred Cubic	{110}{112}{123} etc.	[111]	{112}	[111]	23, 24, 25
Tungsten	"	{112}	"			45
Copper-zinc β	"	{110}	"			22
Bismuth	Rhombohedral	{111}?	[101]?	{111}	[101]	26, 27
					[110]	28, 29, 30
Antimony	"	Deformation by slip doubtful				29, 31
Tellurium	Rhombohedral	{1010}	[1010]	{0001}	[1010]	32
Cadmium	Hexagonal close-packed	{0001}	[1010]	{0001}	[1010]	33
Zinc	"					34-40
Magnesium	"					41, 42, 43
Zinc-cadmium	"					37
Zinc-tin	"					37
Magnesium-Alloys						42, 43
Tin	Tetragonal Diamond type	{110} chiefly {100}	[100] [101] [111]	{110} {100}	[001] [100] [111] [101]	44, 46
		{101} less {121} common	[101]			

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Reference to the table shows that in most cases the slip-plane is that crystal plane which has the highest atomic density. Similarly, the direction of slip is along the line of greatest atomic density on this plane. Even where the plane of slip is not clearly defined, as in the body-centred cubic crystals, there is no ambiguity about the direction of slip. Antimony and bismuth are exceptional in that the whole of the deformation probably takes place by the formation of twins.† This is not so remarkable if it is remembered that the planes of highest atomic density, on account of the symmetry, do not contain the directions of greatest atomic density. In all cases where the distortion has been worked out completely, the particular planes or combination of planes and directions of slip are determined by stress distribution. This aspect of the problem will be dealt with in the next chapter.

† See Chapter IV.

III

CALCULATION OF FORCES REQUIRED TO PRODUCE DEFORMATION OF CRYSTALS

WHEN a load is applied to a metal test-piece there is no permanent deformation until a certain critical value is reached. As soon as this point is exceeded the metal will either fracture completely, i.e. it is brittle, or will deform by one of the methods already prescribed. It is necessary to consider the stresses normal and tangential to the slip-plane.

The tangential component of shear is given by the formula

$$S = \frac{T}{A} \cos \theta \sin \theta \cos \eta, \quad (1)$$

and the component normal to the plane

$$N = \frac{T}{A} \cos^2 \theta, \quad (2)$$

where

T = total load on the test-piece (either in tension or compression),

A = original cross-sectional area of test-piece,

θ = angle between axis of test-piece and the normal to slip-plane,

η = angle between direction of slip in the slip-plane and the direction of maximum slope, i.e. the projection of the axis on to the slip-plane.

It is evident, therefore, that the geometry of the crystal, which includes the relationship of the planes to each other, i.e. the crystal symmetry, and the orientation of the crystal relative to the stress, will have an important bearing on the experimental values obtained in testing.

There are eight octahedral planes in the cubic crystal, of which four are parallel to the other four, so that from the point of view of distortion there are four possible slip-planes in metals having a face-centred cubic lattice. It is evident from a consideration of the formulae (1) and (2) that there will generally be one plane on which the shear stress is greater than on the other three because of its inclination to the axis. That plane will function as the slip-plane.

Similarly there are three possible directions of slip, corresponding to the three $[110]$ directions in every $\{111\}$ plane. But the shear stress

is greatest in the direction nearest to the direction of maximum slope, i.e. where η is small. The same considerations apply to zinc and other crystals of hexagonal symmetry. There, the problem is simplified in that there is not the same choice of slip-plane, although there is still a choice of direction of slip. Iron and other metals having body-centred cubic lattices present peculiar features which have been referred to already. The slip-plane is a combination of crystal planes determined chiefly by stress distribution.

As all the crystals follow this law, it is possible to predict the slip-plane and direction of slip of any crystal whose orientation is known.† It has already been described (p. 19) how (in the case of cubic crystals) a stereographic figure can be obtained in which a point P , representing the axis of the specimen, can be put into one of 48 similar spherical triangles (Fig. 7). By the suitable choice of the $[100]$ axis to put into the centre, P always falls into the same triangle. Now if, for every position of P , the value of $\cos \theta$, $\sin \theta$, $\cos \eta$ is calculated for the 4 possible slip-planes and the 3 possible directions of slip, it will be found that the sphere is divided into areas, in each of which the shear stress on one particular $\{111\}$ plane and in one $[110]$ direction is greater than on any other plane, or in any other direction. These areas correspond to the spherical triangles of Fig. 7. In other words, if the axis P lies in a particular triangle, it will deform by slip on one particular plane and in one particular direction, and the indices of the slip-plane (on top) and those of the direction of slip (underneath) are marked in each triangle in Fig. 14. Actual measurements of θ and η can be made by using a stereographic net, as soon as the orientation is known from X-ray measurements. If the axes are rotated so that a $[100]$ axis comes into the centre of the projection and the other axes are moved correspondingly, the figure may be placed on a diagram similar to Fig. 14, and from the position of P the slip-plane and direction of slip are known immediately. Reversing the process gives the positions of these planes or directions in the test-piece.

It has been mentioned already that slip will take place on two $\{111\}$ planes simultaneously when, by rotation of the crystal axes during the extension, two such planes make equal angles with the axis. Fig. 14 helps to explain this further. As soon as the axis P which is moving along the great circle towards the pole of the (110) plane, the direction of slip, crosses the boundary of one of these triangles, the

† G. I. Taylor, C. F. Elam, *Proc. Roy. Soc. A*, **108** (1925), 28.

shear stress on the first slip-plane becomes less than on another similarly situated, and this second plane then starts to function. Actually, it is found that slip tends to go on somewhat longer on the first plane than would be expected, and this is particularly marked in certain alloy crystals (cf. Figs. 8-11, Chapter II).

The fact that face-centred cubic crystals were found to deform in

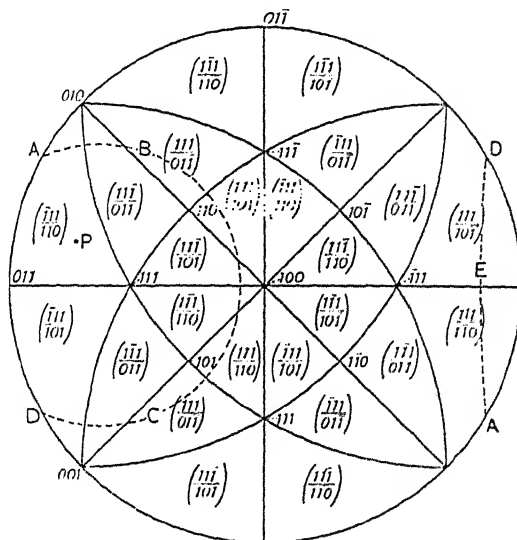


FIG. 14. Stereographic diagram for predicting slip-plane and direction of slip from knowledge of orientation of crystal.

the way described and to follow definite laws, enabled von Göler and Sachs† to construct charts for predicting the extension, strength, etc., of aluminium crystals, from a knowledge of the original orientation relative to the axis of the test-piece. These charts are based on the relationships between the position of the slip-plane and the direction of slip, before and after extension or compression; the amount of extension; change in cross-sectional area; etc. There was in every case, good agreement between calculated values and observed results.‡ (Cf. p. 38.)

The case of iron is somewhat different. Here, the direction of slip coincides with a crystallographic direction, i.e. the normal to a $\{111\}$ plane, but not the slip-plane, which varies between a $\{110\}$ and $\{112\}$

† von Göler, G. Sachs, *Z. Physik*, **41** (1927), 103.

‡ G. Karnop, G. Sachs, *Z. Physik*, **41** (1927), 116.

plane. The slip-plane, however, is definitely related to the distribution of stress.

Further light was thrown on this problem by the study of β brass crystals.† These also have a body-centred cubic lattice, and the distortion was of the same type as in iron. In this case there was definite evidence that the original orientation influenced the position of the slip-plane, and that the resistance to shear varies as the plane of slip rotates about the direction of slip. If the resistance to shear were the same for all possible planes passing through the direction of slip, the actual slip-plane would be that one in which the direction of slip, i.e. [111] direction, was also its direction of greatest slope. This direction corresponds to the line SM in all the diagrams. Comparing iron and β brass it was found that the pole of the slip-plane for iron was near that of the $\{112\}$ plane, for β brass near that of the $\{110\}$ plane, both of which are perpendicular to the direction of slip in the slip-plane and, except in two iron crystals, the points never coincided with either. This can only mean that the resistance to shear on the $\{112\}$ and $\{110\}$ planes is different and in the opposite direction in the case of iron and β brass respectively.

Another calculation of the resistance to shear showed the influence of orientation.

Let

ξ = inclination of axis of specimen to direction of slip,

χ = angle between the normal to the crystal plane (101) and the plane containing the direction of slip and the axis of the specimen,

ψ = angle between plane of slip and crystal plane (101),

P = total pressure on a compression specimen of area a , or total pull in a tension specimen of cross-section a .

Then the component of shearing force parallel to the direction of slip is:

$$F = \left(\frac{P}{a}\right) \cos \xi \sin \xi \cos(\chi - \psi). \quad (3)$$

The slip-plane would be expected to be that plane which, as ψ was varied, presented the least resistance to shear, i.e. which had the lowest value F .

From (3) $F/\{\cos \xi \sin \xi \cos(\chi - \psi)\}$ is a minimum, so that if F is

† G. I. Taylor, *Proc. Roy. Soc. A*, **118** (1928), 1.

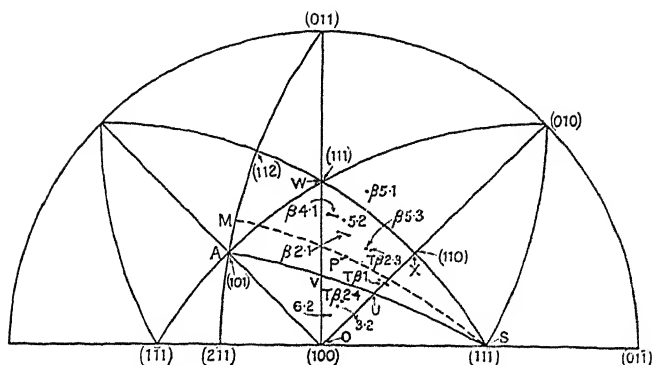


FIG. 15. Positions of axes of specimens of β brass relative to crystal axes.

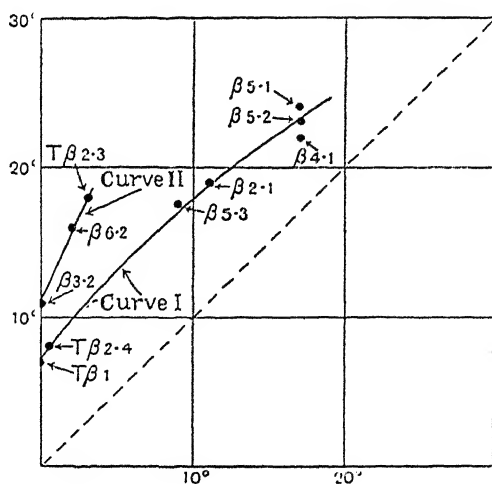


FIG. 16. Curves showing relationship between angle between normal to crystal plane (101) and plane containing the direction of slip and axis of specimen, and angle between plane of slip and plane (101) (ψ).

regarded as a known function of ψ , independent of ξ (which is constant for any given crystal) the equation for determining χ may be written:

$$1 \frac{dF}{d\psi} \dots \psi. \quad (4)$$

Conversely, if the relationship between χ and ψ is known, the relationship between F and ψ can be determined.

In the following Fig. 16 numerical values for χ and ψ are plotted, which were obtained from the crystals whose original positions

are shown in Fig. 15. They fall on two curves. Points on the upper one correspond to 1 tensile specimen and 2 compression; on the lower to 1 tensile and 4 compression. Reference to Fig. 15 shows that this division corresponds to the division of the triangle $W XO$ by the great circle through the pole of the $(11\bar{1})$ plane, i.e. the direction of slip, and the (101) plane. The distortion is crystallographically the same

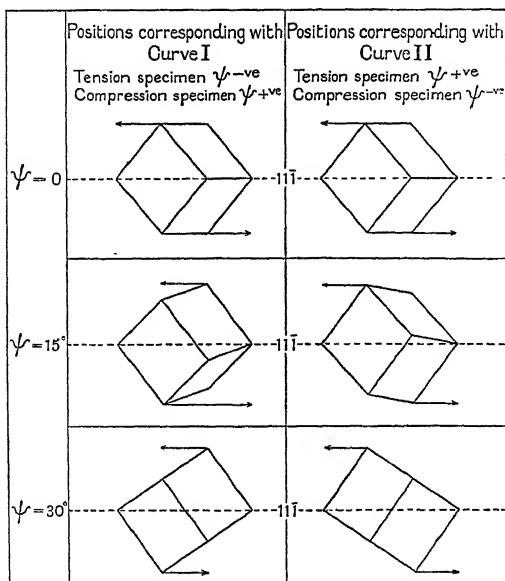


FIG. 17. Diagram illustrating crystallographic difference between shearing in opposite directions.

whichever part of the triangle a point is in, but the *sense* of the direction of slip is opposite. It is for this reason that we find the tensile specimen whose axis falls in $XUVW$, corresponding to compression specimens in this area, and vice versa. Fig. 17 shows how this can arise. On the left are three positions of the crystal relative to the direction of slip (dotted line) for points on curve I (lower). On the right the corresponding positions for points on curve II. The arrows represent the direction and sense of slip. It may be pointed out that $\psi = 0^\circ$ corresponds to the (101) plane, and $\psi = 30^\circ$ to the (112) plane.

By similar methods from formula (4) the variation of resistance to shear with orientation of slip-plane can be calculated. The results indicate a minimum when $\psi = 0$, i.e. when the slip-plane is the (101) plane. In these results, there is indirect confirmation that the

distortion of β brass crystals takes place by means of slip on one or more planes of the type $\{110\}$ (p. 24).

Some of the geometrical considerations outlined above are confirmed by experiment. Others are not so easily demonstrated owing to the influence of greater changes which mask small variations in orientation. The influence of inclination of slip-plane and direction

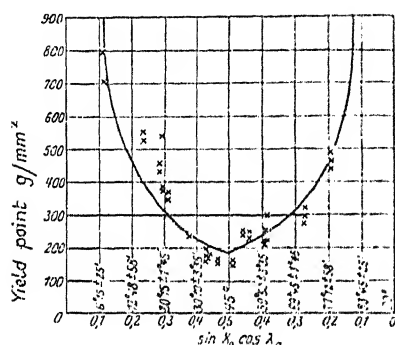


FIG. 18. Diagram illustrating variation in value of yield-points with orientation of zinc crystals.

of slip on the forces required to distort a crystal has been clearly demonstrated. Let us first consider the simpler case of zinc and cadmium crystals which have one slip-plane. The load required to extend 0.2 per cent. crystals, whose slip-plane and direction of slip vary relative to the axis, has been measured and the shear stresses calculated. The results for zinc are shown in a paper by Rosbaud and Schmid† (Fig. 18).

The yield-point, as defined above, varies according to the inclination of the slip-plane and direction of slip to the axis. The calculated stress normal to the plane also varies from 8 gm./mm.², when the angles are small, to 463 gm./mm.² when they are large. The shear stress, on the contrary, is nearly constant at about 94 gm./mm.² Similar results have been obtained with cadmium,‡ tellurium,§ bismuth,|| etc.

Polanyi and Schmid†† have also shown that the yield-points of zinc and tin crystals are not altered by hydrostatic pressures up to 40 atm. The evidence is therefore against any influence of the normal stress on the resistance to shear.

The determination of the critical normal stress for different crystal planes and directions has enabled 'Festigkeitskörper' to be constructed for a number of metal crystals. These are produced in the following way. From a point on a line representing the middle of

† P. Rosbaud, E. Schmid, *Z. Physik*, **32** (1925), 197. See also E. Schmid, *Proc. 1st Int. Mech. Sci. Cong.*, Delft (1924), 341.

‡ W. Boas, E. Schmid, *Z. Physik*, **54** (1929), 16.

§ E. Schmid, G. Wassermann, *Z. Physik*, **46** (1928), 653.

|| E. Schmid, *Z. Physik*, **39** (1926), 359.

†† M. Polanyi and E. Schmid, *Z. Physik*, **16** (1923), 336.

the figures, radii vectores are drawn whose lengths represent the magnitude of the stress and whose directions are the corresponding directions in the crystal to which the experimental results refer. Similar figures can be drawn to represent elasticity, shear stresses, extensions, etc.†

As soon as the yield-point has been exceeded two new factors must be considered:

1. By the process of slip, the orientation of the slip-plane relative to the axis is continually changing, i.e. θ becomes larger (see p. 19). Therefore the shear stress on the slip-plane will increase.

2. The metal hardens.

The increase in shear stress on the slip-plane due to change of orientation is evident from the experiments described in the previous section. If there were no increased resistance to shear to counter-balance this, the metal would fail, without any further increase in load, on the original slip-plane or planes. The extreme uniformity of the distortion and the shape of the stress-strain diagrams for certain single-crystal test-pieces show that, when slip begins on a given slip-plane, a hardening process also begins, which tends to hinder further slip. Distortion will then take place by the use of other planes and these in turn will be hardened and cease to function under a given load. As soon as the load is increased the process will begin again.

If the shear stress on the slip-plane for all stages of the distortion is plotted against extension for a number of aluminium crystals, the curve is of the type shown in Fig. 19 and it will be seen that the values for different crystals are very close. The agreement is not so good in the case of zinc.‡ The explanation probably lies in the very much greater influence of the original inclination of the slip-plane in zinc, compared with aluminium and other face-centred cubic metals.

In Fig. 20§ a comparison is made between the curve (I) obtained from a single zinc crystal and that (II) which would be obtained, if the increase in shear stress were due to changes of orientation only. The part played by change in orientation is relatively small.

A more accurate picture of the process of hardening can be obtained if the shear stress, S , is plotted against amount of shear, s , at unit distance parallel to the slip-plane.||

† E. Schmid, loc. cit. Karnop, Sachs, *Z. Physik*, **41** (1927), 116.

‡ E. Schmid, *Z. Physik*, **40** (1926), 54.

§ E. Schmid, *Z. Physik*, **22** (1924), 328.

|| G. I. Taylor, *Proc. Roy. Soc. A*, **116** (1927), 16.

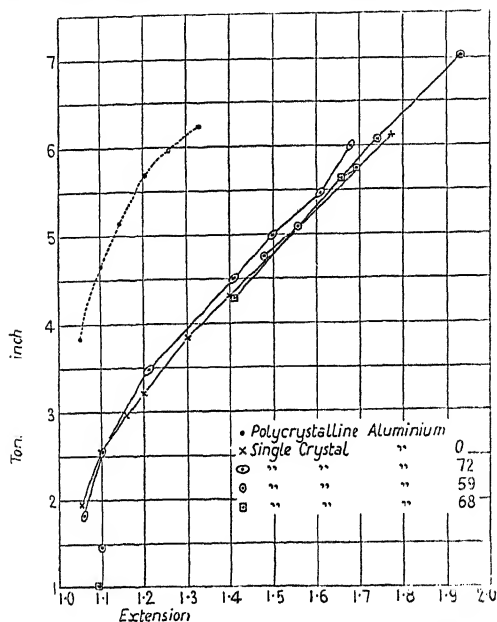
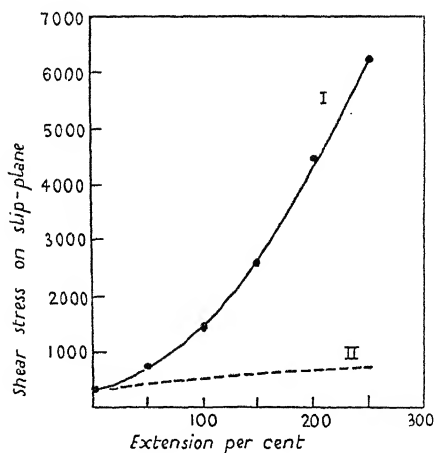


FIG. 19. Stress-strain curves for aluminium crystals.

FIG. 20. I = measured,
II = calculated for change of orientation only.

In Fig. 6, Chapter II, if $ABCD$ is the undistorted position of a rectangular block of the crystal, the distorted position is $AB'C'D$, where AD is the slip-plane. The amount of shear is defined as

BB'/BA , that is, $\tan BAB'$. The displacement of B , i.e. BB' , is thus $AB \tan BAB'$.

The formulae for calculating the amount of shear in compression and tension specimens are given in Taylor's paper, and the problem is dealt with in a similar way by von Göler and Sachs.[†] Rosbaud and Schmid,[‡] in the paper already referred to, also took into consideration the displacement parallel to the slip-plane.

Take rectangular coordinates x, y, z so that the plane $z = 0$ is parallel to the plane of slip, OX is the direction of slip, and O is in the upper surface of a compression specimen. The spherical polar coordinates of the normal to the slip-plane are θ and η , as defined previously. The relations between θ, η and x, y, z are

$$x = r \sin \theta \cos \eta,$$

$$y = r \sin \theta \sin \eta,$$

$$z = r \cos \theta,$$

where r is the distance of the point considered from the origin.

Let θ_0, η_0 be the values of θ and η before distortion. After the material has been subjected to a shear s parallel to the plane $z = 0$, and in the direction of the axis of x , the coordinates are

$$r \sin \theta_0 \cos \eta_0 + s r \cos \theta_0,$$

$$r \sin \theta_0 \sin \eta_0,$$

$$r \cos \theta_0.$$

ϵ , the ratio of the final to the initial length is given by

$$\epsilon^2 = (\sin \theta_0 \cos \eta_0 + s \cos \theta_0)^2 + \sin^2 \theta_0 \sin^2 \eta_0 + \cos^2 \theta_0$$

$$\text{or} \quad \epsilon^2 = 1 + 2s \cos \theta_0 \sin \theta_0 \cos \eta_0 + s^2 \cos^2 \theta_0,$$

from which s can be calculated.

The shearing stress $S = (T/a) \sin \theta \cos \theta \cos \eta$, where T is the total force and a is the area of cross-section of the test-piece. The formulae connecting θ, η with θ_0, η_0 are

$$\left. \begin{aligned} \epsilon \cos \theta &= \cos \theta_0 \\ \epsilon \sin \theta \sin \eta &= \sin \theta_0 \sin \eta_0 \end{aligned} \right\},$$

so that $S = (T/a_0) \cos \theta_0 \{1 - \epsilon^{-2} (\cos^2 \theta_0 + \sin^2 \theta_0 \sin^2 \eta_0)\}^{\frac{1}{2}}$.

von Göler and Sachs use a similar method. They choose a system

[†] von Göler and G. Sachs, *Z. Physik*, **41** (1927), 103.

[‡] P. Rosbaud and E. Schmid, *Z. Physik*, **32** (1925), 197.

of coordinates so that the ξ -axis is parallel to the normal to the slip-plane, the η -axis is parallel to the direction of slip, and the ζ -axis at right angles to both. ϕ is the angle which the axis of the test-piece makes with the ξ -axis, and ρ the angle which the axis makes with the direction of slip, i.e. the η -axis. The shear t , which is equivalent to Taylor's s , is given as follows,

$$t = \frac{\cos \rho}{\cos \phi} \frac{\cos \rho_0}{\cos \zeta}$$

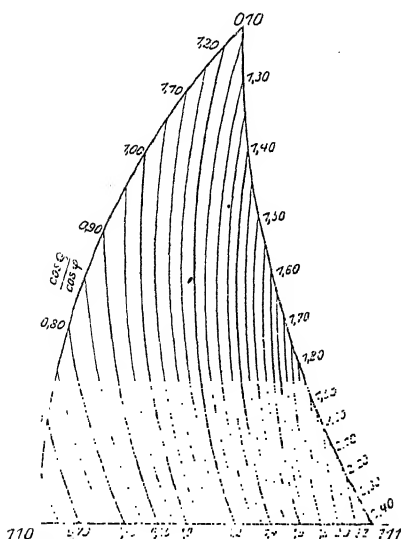


Chart for calculating amount of shear for slipping on one plane.

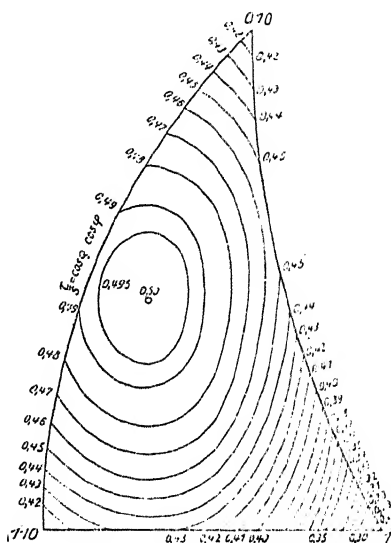


Chart for calculating shear stress.

Fig. 21.

These are the formulae for gliding on one plane. A similar method of calculation has been applied to double gliding. For details, the reader is referred to the original papers, but the two methods have been described here briefly in order to explain the different methods of presenting the results. In most of the English work the values for s are given and plotted direct. In the papers of Sachs and his collaborators, t is generally referred to as the percentage of $\cos \rho / \cos \phi$.

von Göler and Sachs have constructed charts similar to those described on p. 30 for calculating these values. Examples are shown in Fig. 21.

The method of plotting shear stress against amount of shear has the advantage that tests in tension and compression can be compared.

Curves obtained for a number of aluminium crystals in tension and compression are shown in Fig. 22.

The fact that the points for both tension and compression specimens lie on the same curve means that the resistance to slip is identical, whether the component of force normal to the slip-plane is a tensile or a compressive one. Moreover, it confirms Schmid's results described on p. 34, viz, that the resistance to shear parallel to the slip-plane is independent of the stress normal to the plane.

Schmid† has obtained similar curves for zinc crystals, Fig. 23.

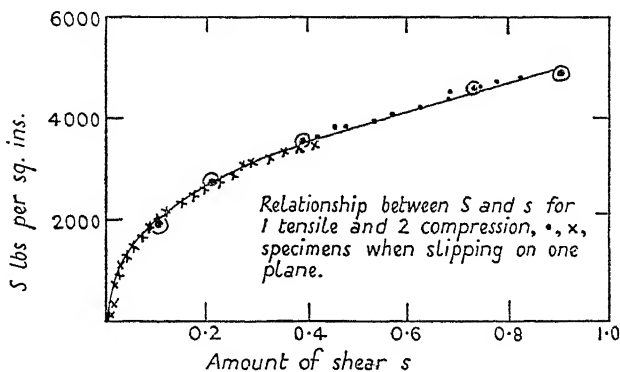


FIG. 22.

Here again, S increases with the amount of shear. Other examples will be found in Chapter VI on alloys.

The stress analysis of single-crystal torsion specimens is referred to in the section on repeated stresses (p. 97), as it has been used chiefly in that connexion by Gough, Hanson, Wright, etc., who developed the method of testing and calculation.

To sum up: the initial strength of a crystal depends on its orientation with regard to the stress, although the shear stress on the slip-plane has a constant critical value. As soon as plastic deformation has begun, the increase in strength is much greater than can be explained by change of orientation only, and seems to be directly related to the amount of shear and to the actual process of gliding. Theoretical considerations will be dealt with later.

Slip-bands are not usually so clear and well-defined that a single band can be traced round a crystal bar or on two faces of a square specimen. It is therefore difficult to count the bands or to compare

† E. Schmid, *Z. Physik*, 40 (1926), 54.

extension with the number produced. An attempt has been made by Yamaguchi† to correlate shear stress on the slip-plane with the number of slip-bands appearing on the surface of aluminium crystals when these are subjected to tension. The number of slip-bands per unit length in the direction normal to the slip-plane, N' , is plotted against shear stress on the slip-plane at different stages of the test in

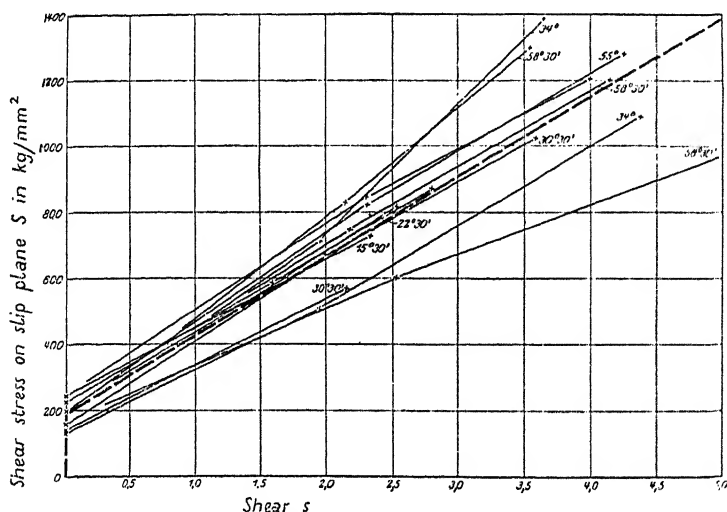


FIG. 23. The angles refer to position of basal plane in original crystal.

Fig. 24. The points lie on a straight line. If the specimen be polished at intervals—i.e. the slip-bands already present being removed—the curves are similar. The number of slip-planes appear to be directly related to the shear stress:

$$N' = a(S - S_0),$$

where

N' = number of slip-bands per unit length normal to the slip-plane,

S = shear stress,

S_0 = shear stress when the first slip-bands appear,

a = proportionality constant depending on material, etc.

The relationship between shear stress, S , and amount of shear, s , is found to be by experiment approximately

$$s = 0.015S^{2.5}.$$

† K. Yamaguchi, *Sci. Papers Inst. Phys. Chem. Res. Tokyo*, 8 (1928), 289.

This may be compared with the figure given by von Göler and

Assuming that the whole of the distortion is made up of slip on the slip-planes and that the amount of slip is equal on every plane, it is

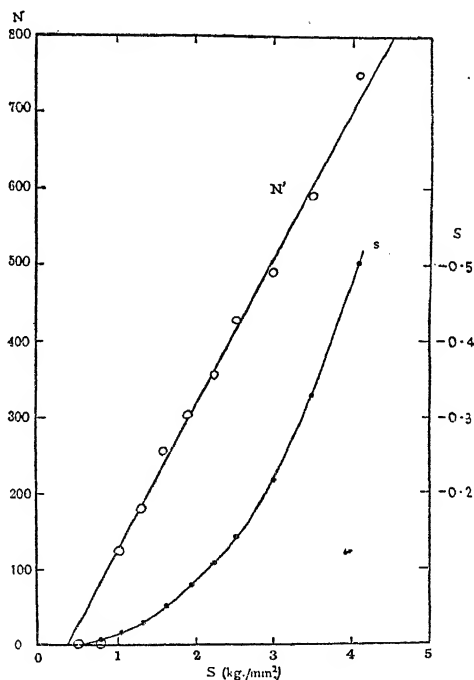


FIG. 24. Diagram illustrating connexion between number of slip-bands, shear stress S , and amount of shear s .

possible to calculate the displacement on each plane. Yamaguchi gives a figure of $70 \mu\mu$, and this value is consistent with the limiting value of differences of level as seen under the microscope.

† von Göler, G. Sachs, *Z. techn. Phys.* 8 (1927), 586.

IV

DEFORMATION BY THE FORMATION OF TWINS

Two crystals or two parts of the same crystal are said to be twinned when they bear certain definite crystallographic relations to one another, in accordance with well-defined laws. The most common conception is of a crystal cut in half along a particular plane, the twin plane, and one half rotated 180° on this plane. The twin plane and also the axis about which one half can be considered as being rotated are thus common to both parts.

It must not be supposed that the two parts of a twin are always united along their twin plane. The composition plane (as it is called) may or may not be the twin plane. It may be another crystal plane, or even an arbitrary plane having no direct relationship to the crystal structure, but in a true twin there is always a *possible* crystal plane upon which such a rotation can be supposed to have taken place.

Although twinning is most often associated with a possible rotation, there are also twins which are only reflection twins, i.e. one part of the twin is the mirror image of the other part in the twin plane.† In such forms, rotation on any plane and about any axis does not make the two parts alike.

It is important to realize that there may be two kinds of twins: (1) those produced by deposition from the liquid and growth; (2) those produced by mechanical deformation. We are only concerned with the latter.

Deformation by gliding, or slipping, described in Chapter II, can never lead to twinning. Comparatively large blocks or layers of the crystal slide over each other, but do not change their relative orientation in so doing, as long as the distortion is uniform and the glide-planes remain parallel. Moreover, the displacement along the slip-planes, provided it is a multiple of the distance between atoms in the direction of shear, depends on the amount of deformation and varies considerably. In order to produce twinning and to change the relative orientation without change of space lattice, it is necessary to move every plane of atoms in the lattice a definite amount. The amount moved must be less than the distance between neighbouring atoms in the direction of shear. The plane which does not change

† 'Rotation' twins are also usually reflection twins.

during the process may be called the glide-plane, but it would be more correct to differentiate between gliding and twinning, and to refer to it as the twin plane. In both cases the deformation may be described as a shear parallel to the glide or twin plane, and in a given direction. The difference between gliding and twinning may be shown by means of diagrams (Fig. 25).

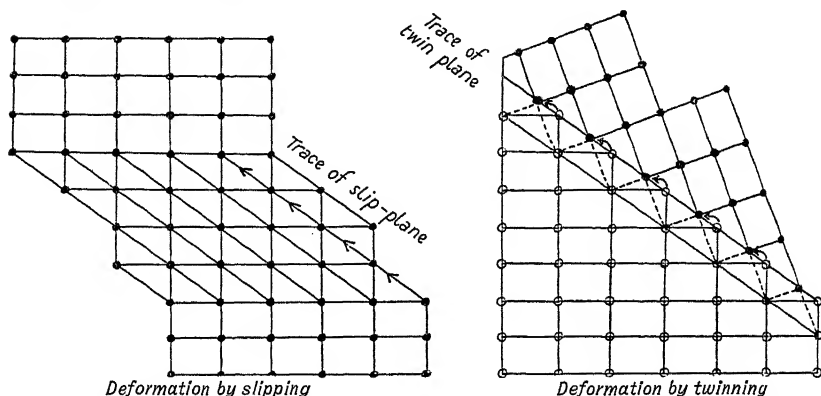


FIG. 25. Deformation by slipping and twinning. In both cases the plane upon which movement takes place is the same.

In both cases the amount of shear may be defined as being equal to $\tan BAB'$ (cf. Fig. 6, p. 18), but the processes are essentially different. Twinning is described by Mügge as 'einfache Schiebung', as opposed to ordinary gliding which is generally referred to as translation. He† investigated a large number of minerals and also zinc and iron.‡ Since then, twins caused by mechanical deformation have been described in a number of metals and their true character established by means of X-rays.

The reasons why twins are formed in some cases and not in others and the mechanism of their formation are unknown. They frequently facilitate deformation by gliding by producing a slip-plane at a more favourable angle than that already functioning. This is particularly noticeable in zinc and cadmium, which normally have only one available slip-plane, the basal plane of the hexagonal lattice. In section (1) is described how the distortion of a zinc crystal takes place. In their original experiments, Mark, Polanyi, and Schmid§ found that towards the end of the extension, i.e. when the slip-plane made a small angle, from 4° to 7° , with the axis of the wire, there was a

† O. Mügge, *Neues Jahrb. Mineral.* **2** (1899), 55.

‡ O. Mügge, *Z. anorg. Chem.* **121** (1922), 68.

§ Cf. p. 13.

sudden thinning down of the section ('Nachdehnung') leading to further extension and fracture. In this region, slip-bands appeared at approximately 90° to the original glide-plane, and they concluded that slip was taking place on one of the planes of the prism, which is perpendicular to the basal plane. It was subsequently shown by Mathewson and Phillips,[†] and Schmid and Wassermann[‡] that the change in the deformation was due to twinning on a pyramidal plane of form $\{10\bar{1}2\}$. This brings the basal plane of the twin into such a position that it makes an angle of only $4^\circ 5'$ with the original prism plane. The slip-bands which appeared to belong to prismatic planes were really basal planes in the twinned position. This was proved by means of external and X-ray measurements by the workers referred to, and their results were also confirmed by fatigue tests on zinc crystals (see Chap. VII).

The amount of shear can also be calculated, in the same way as for simple gliding. Schmid and Wassermann have done so in the following manner. In Fig. 26 let K_1 represent the trace of the plane through the glide-plane or twin plane and the direction of shear, in the first position, and S the shear at unit distance from the glide-plane. In the second position K_2 is inclined to K_1 at an angle $2\phi_1$. From this the following relation is deduced:

$$\tan 2\phi_1 = \frac{2}{S}. \quad (1)$$

If the measured angles in the case of zinc are put in Fig. 26, assuming that the movement suggested by Mathewson and Phillips is correct, one can calculate S :

$$S = \frac{\sin 4^\circ 5'}{\sin 42^\circ 57' \cdot 5' \sin 47^\circ 2' \cdot 5'} = 0.1428. \quad (2)$$

Substituting in (1) the angle between K_1 and K_2 can be calculated. This proves to be $85^\circ 55'$, which is the calculated angle between two planes of indices $(10\bar{1}2)$ and $(10\bar{1}\bar{2})$.

In Fig. 27 if ϕ = angle between axis of rod or crystal and twinning plane, and α = angle between the projection of the axis on the twinning plane and the direction of gliding, the increase in length may be expressed thus:

$$\frac{l_1}{l_0} = \sqrt{(1 + 2S \cos \alpha \sin \phi \cos \phi + S^2 \sin^2 \phi)}.$$

[†] C. H. Mathewson and A. J. Phillips, *Proc. Amer. Inst. Min. Met. Eng.* (1927), 143.

[‡] E. Schmid, G. Wassermann, *Z. Physik*, 48 (1928), 370.

Metal		Twin plane		Amount of Shear S
		K_1	K_2	
Cu } α brass }	Cubic face-centred	(111)		
α Fe	Cubic body-centred	(112)	(112)	0.7072
Sn	Tetragonal	(331)	(111)	0.1197
As }	Rhombohedral	{(011)	(100)	0.2562
Sb }		{(011)	(100)	0.1463
Bi }		{(011)	(100)	0.1176
Zn }	Hexagonal	(10 $\bar{1}$ 2)	(10 $\bar{1}$ 2)	0.1428
Be }		{(10 $\bar{1}$ 2)		
Mg }				
Cd }				

when twinning takes place is $S = \sin 12^\circ \cos 12^\circ Z = 0.20Z$ (if Z = total load on the crystal). After twinning, the basal plane is approximately 60° to the axis. Hence the shear stress is doubled, i.e. $\sin 60^\circ \cos 60^\circ = 0.43$. On the other hand, the shear stress on the new basal plane is considerably less.

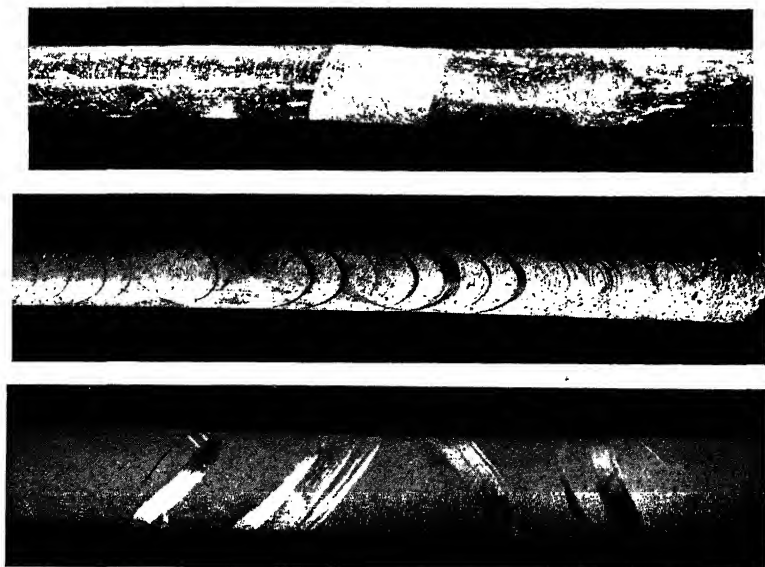
The work on zinc was confirmed by similar results for cadmium.† It was also found in cadmium, that the greater the hardening during the initial slip on the basal plane, the less the increase in the shear stress during twinning. Twins‡ of the same form are produced in magnesium crystals, but they occur at the beginning of the deformation. Twinning on planes of the type $\{101\bar{1}\}$ are also reported. Photographs of twin crystals are shown on Plate III, Figs. 1-3.

Some interesting observations on twin formation in zinc were made by Gough and Cox,§ when they carried out alternating torsion tests on zinc crystals. A description of their experiments and results will be found more fully described in Chapter VII. Under the conditions of their tests, all six possible directions of twinning on $\{1012\}$ planes were observed. Only one pair of complementary twinning planes occurred in association with one slip direction. The operative twinning planes changed with the change in operative slip direction. In any operative slip direction, the twinning planes containing that direction did not appear. They came to the conclusion that the formation of twins might be related to slip on the basal plane but that they could not be produced by means of slip on the pyramidal or twin planes themselves.

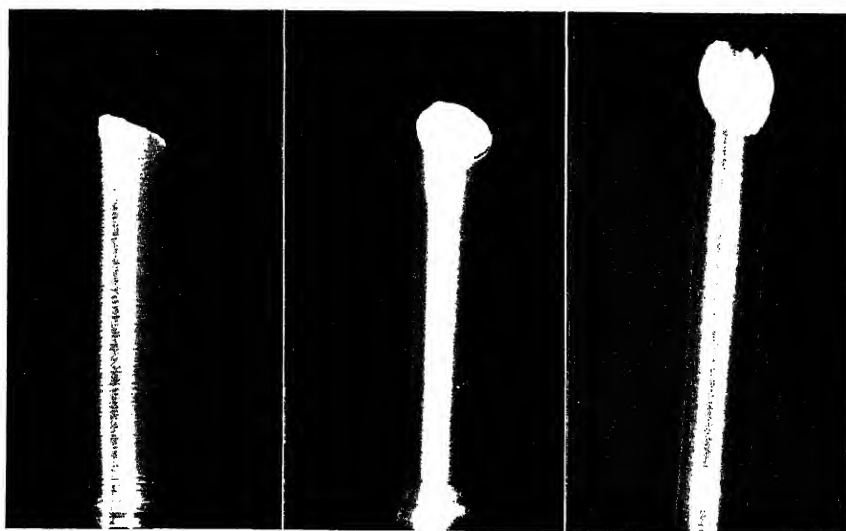
† W. Boas, E. Schmid, *Z. Physik*, **54** (1929), 16.

‡ E. Schmid, *Z. Elektrochem.* **37** (1931), 447.

§ H. J. Gough, H. T. Cox, *Proc. Roy. Soc. A*, **123** (1929), 143; **127** (1930), 453.



FIGS. 1, 2, and 3. Photographs of twin formation in zinc, bismuth, and cadmium



FIGS. 4, 5, and 6. Aluminium-zinc alloy crystals

By similar methods the same workers† determined the twinning planes in antimony crystals. These proved to be of the $\{011\}$ type and all three possible forms were observed.

Twinning in bismuth occurs on planes of type $\{110\}$, according to Mügge.‡ This has been corroborated by means of alternating torsional tests by Gough and Cox.§ No slip-bands were observed in these crystals so that, as in the case of antimony, the whole of the distortion seems to take place by means of twin-formation.

Twinning in crystals of face-centred cubic lattice

Metals having a face-centred cubic lattice, i.e. gold, silver, and copper, have been found in the native state with well-marked twins of the spinel type, i.e. twinned on an octahedral plane. Twins of this type have also been described in annealed specimens of aluminium,|| silver,|| copper,|| and brass,†† and many other metals have structures which resemble twins but have not been proved by means of X-rays, etc., to be so. There is, however, no record of such twins being formed by direct mechanical means.

Structures resembling twins, however, might be expected to arise during distortion by slip, if the crystal were in such a position that two or more slip-planes were equally inclined to the axis. Several crystals have been obtained in which double slipping, such as is described in Chapter II, took place from the beginning of the extension because two octahedral planes were equally inclined to the axis. If in such a crystal, owing to some slight irregularity, slip took place on one octahedral plane in one part and on the other octahedral plane in another, the crystal might, superficially, appear to be twinned. Such an example was met with in a copper alloy crystal containing 5 per cent. Al.‡‡ The original orientation was close to the position for double slip but not exactly so. In alloys (Chap. VI) there is frequently a tendency for slip to go on longer on the slip-plane that is functioning at the moment than would be expected from shear stress considerations and simultaneous double slipping does not occur. In the case under consideration, the top and bottom of the test-piece behaved normally, but in the middle, slip occurred on the octahedral

† H. J. Gough, H. T. Cox, *Proc. Roy. Soc. A*, **127** (1930), 431.

‡ O. Mügge, *Neues Jahrb. Mineral.* **1** (1886), 183.

§ H. J. Gough, H. L. Cox, *J. Inst. Metals*, **38** (1932), 227.

|| C. F. Elam, *Proc. Roy. Soc. A*, **121** (1928), 237.

†† A. J. Phillips, *Proc. Amer. Inst. Min. Met. Eng. Techn. Pub.*, No. 56, 1928.

‡‡ C. F. Elam, *Proc. Roy. Soc. A*, **116** (1927), 695.

plane and in the direction which would be expected to act, when the position for double slipping was reached. Both parts then proceeded normally and when fracture occurred, the orientation of each part relative to the axis was almost the same, if one was regarded from above and one from below.

Another example may be quoted,[†] where a somewhat similar type of deformation must have occurred, giving a structure which fulfilled all the necessary requirements[‡] of a twin. Fig. 5, Plate II, is a photograph of a section through an aluminium crystal test-piece. The original section was round and the non-uniformity developed during extension. The original orientation was unknown, but a determination of the crystal axes in the two parts, *A* and *B* of the photograph, showed that a [112] direction coincided with the axis in both and the two were so orientated that they formed mirror images of one another in their plane of union. This plane proved to be a plane of form {113}. If slip on two neighbouring parts is confined to one plane, it is necessary, in order that the parts shall fit, that (1) the surface of separation should contain the axis of the specimen, (2) the crystal axes on the two sides should be in the position of mirror images in this plane, (3) the amount of slipping should be equal in both parts. All these conditions are fulfilled. With regard to the original orientation of the crystal relative to the axis there are two possibilities. The axis might lie in (1) a cube plane, (2) a dodecahedral plane. No. 2 corresponds to the conditions for double slipping in normal crystals and is not likely to lead to twinning. Taylor[‡] was able to show from geometrical considerations that if the axis lay in a cube plane, a structure such as was actually obtained might be expected and that the two parts would be symmetrical about a {113} plane.§

Twinning in crystals of body-centred cubic lattice

The commonest example of twinning in metals of this lattice is the formation of Neumann bands in iron. They were first recognized by Neumann in meteoric iron—hence the name. It is interesting to note that these bands are only produced when soft iron is subjected to deformation by shock. Slow straining yields slip-bands only. Pfeil||

† C. F. Elam, *Proc. Roy. Soc. A*, **121** (1928), 237.

‡ See Appendix to above paper.

§ Twin-like structures of this type can arise in crystals of any lattice and are common in β brass (body-centred cubic). C. F. Elam, *Nature*, **133** (1934), 723.

|| L. B. Pfeil, *J. Iron Steel Inst. Carnegie Scholarship Mem.* **25** (1926), 319.

has also found that a small preliminary compression or tension (about 0.28 per cent.) was sufficient to prevent the formation of the bands when the specimen was subsequently subjected to impact. Further, this effect could be removed by heating; the greater the deformation, the higher the temperature required as follows:

Compression per cent.	Single Crystals Degrees C.	Finely crystalline Degrees C.
1	95 ± 2.5	95 ± 2.5
5	140 ± 5	130 ± 5
10	160 ± 10	160 ± 10
15	500 ± 25	700 ± 25

Mügge† made measurements on a piece of soft iron, and Osmond and Cartaud‡ cut a specimen with cube faces from a large iron crystal and traced the intersections of the Neumann bands on these faces, and both came to the conclusion, that the bands represented twins on the $\{112\}$ planes of the crystal. This type of twin is common in fluor-spar. Similar measurements by a large number of other investigators have confirmed these conclusions but it was not till much later that definite proof was obtained that the orientation of the metal inside the bands was in fact twinned in respect to the original crystal. Two special methods of attack are available: (1) X-ray photographs, (2) study of etch figures.

(1) The very fineness of the bands makes it difficult to obtain X-ray photographs from isolated bands. Mathewson and Edmonds§ took Laue photographs through a silicon-iron crystal which contained Neumann bands, and from an analysis of the spots were able to show that some of them could be derived from a lattice which represented twinning on two $\{112\}$ planes.

(2) An extensive study of the occurrence and formation of Neumann bands in meteorites has also been made.|| In this case sections were cut in different directions and etched with copper ammonium chloride. This reagent attacks planes of the form $\{110\}$, and from an examination of etch figures in the bands and on the surface of the original crystal it was concluded that the bands represented true twin structures.

† O. Mügge, *Neues Jahrb. Mineral.* **2** (1899), 55.

‡ F. Osmond, G. Cartaud, *J. Iron Steel Inst.* **71** (1906), 444.

§ C. H. Mathewson, G. H. Edmonds, *Proc. Amer. Inst. Min. Met. Eng. Techn. Pub.* No. 139, 1928.

|| S. W. J. Smith, A. A. Dee, J. Young, *Proc. Roy. Soc. A*, **121** (1928), 477.

THE DEFORMATION OF POLYCRYSTALLINE METAL

WE have so far only considered the deformation of single crystals, although, historically, this subject lagged far behind investigations of polycrystalline metals. In the former the change of shape can be measured and completely accounted for on a simple geometrical basis. A similar method is also possible when the individual grains of a normal metal are so small that the material behaves as an isotropic substance. Between the two extremes there are all possible gradations of crystal size. In an aggregate, each grain can no longer deform according to the conditions described in Chapters II and III. It is prevented from doing so by its neighbours, which will probably be orientated at random to the stress. There is thus a tendency for the polycrystalline material to appear harder than the single crystal, particularly during the earlier stages of the deformation, owing to the greater difficulty each crystal experiences in deforming according to the laws already outlined. A similar effect is produced by the presence of a second constituent, as in many alloys. Here, the action is enhanced, if this constituent is itself hard and not easily deformed.

1. General appearance of polycrystalline metal after deformation

When a metal solidifies slowly from the liquid, it does so first by the formation of crystal nuclei and then by the growth of the crystals towards each other until they meet. The junction of one such crystal with its neighbours is generally irregular in outline and there is frequently interlocking. If the cooling has been uniform the individual grains will be of approximately the same size and shape. If one part has been chilled, nuclei will form there first in larger numbers and grow out towards the hotter part of the mould, where the metal solidifies last. Such crystals are very much elongated in one direction.

Crystals that have formed from the melt are usually stable; that is to say, they do not change in shape or size after solidification is complete even with prolonged heat treatment,[†] unless they undergo a polymorphic transformation, or in the case of an alloy, a phase

[†] H. C. H. Carpenter, C. F. Elam, *J. Inst. Metals*, **23** (1920), 83.

change. If such a specimen is hammered, rolled, stretched, or deformed in any way and examined under the microscope, the individual grains will be found to be distorted and elongated following the alteration in shape of the specimen.

These distorted crystals are no longer stable and on heating to a suitable temperature, depending on the amount of deformation, will recrystallize in the solid, the elongated grains being usually replaced by equiaxed ones. At the same time, the physical properties return to their original state. This process is described more fully in Chapter XIV. Figs. 1, 2, and 3, Plate V, are typical examples of the microstructure of a polycrystalline metal in the undistorted and in the distorted state. The extent to which deformation can be carried without obliterating the crystal boundaries is very remarkable. In the extreme case the crystals become like fine threads, hence the term 'Faserstruktur', or like leaves of paper in a book.

Reference has already been made to the appearance of slip-bands on a polished surface, and there is every reason to think that the individual grains follow the same general laws as single crystals of that metal. What happens actually at the boundary is not quite clear. It is a fact, however, that, contrary to what might be expected, the individual grains do not usually pull apart, and under ordinary conditions of test fracture will occur across them rather than between them. This is very beautifully illustrated in some aluminium test-pieces consisting of two crystals such as are shown in Figs. 3 and 4, Plate II. The scratches, which were originally at right angles to each other and parallel with and at 90° to the axis of the test-piece, also show the different extent of the deformation in each crystal and the very complicated distortion which occurs near the boundary. The amount of distortion which has actually taken place at the boundary is very small. In other words, each crystal has prevented the other from deforming. The distance from the boundary, beyond which this influence is no longer felt, will depend on the relative orientations of the neighbouring crystals, but it will be seen that given small crystals completely surrounded by others of differing orientation, the whole crystal volume may be subject to this influence. Figs. 3, 4, Plate II, show clearly how one crystal will deform very much more under a given load than another differently orientated. This is impossible in a crystal aggregate without the grains becoming separated, i.e. it is essential that each grain extends the same amount. In order

to do this most of the crystals must slip on more than one plane, from the beginning of the extension.

Some measurements on the change of orientation of individual crystals in the body of the crystal and in the neighbourhood of the boundaries were made by R. L. Aston.† He used a round aluminium test-piece consisting of one crystal at one end through the whole cross-section and two parallel crystals in the other half. The change of orientation was followed during extension up to 20 per cent. increase in length. The change at the boundary was extremely small and increased towards the body of the crystal. Its effect was felt from 2–5 mm. from the boundary. There was no evidence of shear between adjacent crystals.

Alternating torsion tests‡ carried out with test-pieces consisting of three crystals also showed the tendency of the metal to deform and fracture in the crystal rather than across a crystal boundary.

2. The influence of crystal size on deformation and mechanical properties

It is important to remember when comparing tests of metals of different crystal size, that the cross-sectional area of the test-piece will have some influence on the results. A tensile test-piece 2 cm. in diameter, in which the average crystal size is 2 mm. in diameter will give a different result from one 2 mm. in diameter containing the same-sized crystals. Hardness measurements, such as Brinell, which depend on the measurement of an impression made by a steel ball, are also misleading. If the crystals are large, the impression is never circular,§ owing to the different resistance to deformation of individual crystals, and in the case of single crystals, to variations along different directions in the crystal.

The differences between tensile tests of single crystals and polycrystalline material vary for different metals. One of the most striking examples is that of zinc (Fig. 28). The resistance to shear is, generally speaking, greater in polycrystalline material, although a single crystal may become harder than the normal metal, at the end of the test (cf. Fig. 19). In this case, it is due to a greater degree of work-hardening in the former. A number of workers have carried

† R. L. Aston, *Proc. Cambridge Phil. Soc.* **23** (1925–7), 549.

‡ H. J. Gough, *Trans. Faraday Soc.* **24** (1928), 137. See also H. J. Gough, H. L. Cox, D. G. Sopwith, *J. Inst. Metals*, **52** (1933), 57.

§ H. C. H. Carpenter, C. F. Elam, *Proc. Roy. Soc. A*, **100** (1921), 329.

out experiments in order to correlate crystal size with hardness. Bassett and Davies† measured the Brinell hardness of cartridge brass, and Angus and Summers‡ did the same for copper and copper containing 4.5 per cent. tin. The curves were all similar to Fig. 29.

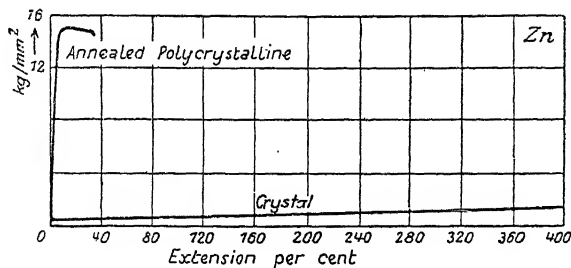


FIG. 28. Comparison between stress-strain curves of single crystal and polycrystalline zinc.

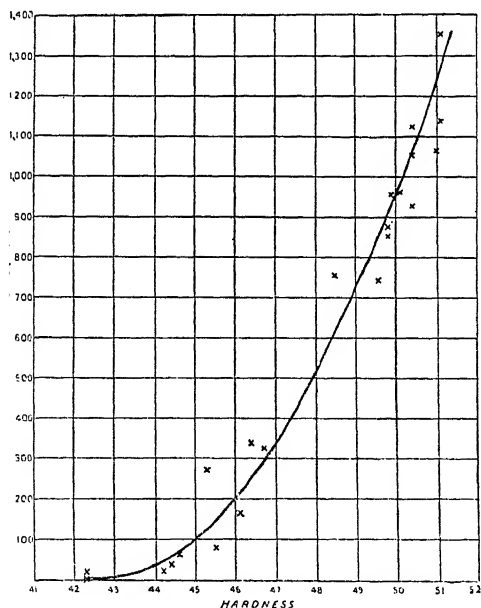


FIG. 29. Change of hardness with grain size.

If the hardness is plotted against amount of grain boundary (estimated as mm^2/mm^3) the relationship is linear.

Ishigaki§ measured the Brinell hardness of iron of variable crystal size,

† Bassett, W. H. Davis, *Trans. Amer. Inst. Min. Met. Eng.* **60** (1919), 428.

‡ H. T. Angus, P. F. Summers, *J. Inst. Metals*, **33** (1925), 115.

§ T. Ishigaki, *Sci. Rep. Tohoku Univ.* **16** (1927), 285.

and Edwards and Pfeil† carried out tensile tests. In both instances the curves show a very rapid rise in the region of large crystals and a more gradual rise from about 100 crystals per sq. mm. onwards. Edwards's and Pfeil's results indicated an optimum crystal size at about 140 crystals per sq. mm. as regards limit of proportionality, tensile strength, etc., but their strips were not treated exactly similarly in order to obtain the different crystal sizes, and their results should not be regarded as conclusive. The yield-point, however, does seem to be affected by crystal size and was absent in their tests on single iron crystals.

The presence of a hard, amorphous, intercrystalline cement has been suggested to account for the greater hardness exhibited by polycrystalline metal. Another suggestion is that the boundaries prevent free deformation by slipping and so enhance the natural resistance to deformation. These theories are discussed in Chapters IX and XV.

3. The structure of deformed polycrystalline metal

The change of orientation produced by extending or compressing metal crystals described in Chapter II resulted in a final stable position of the crystal axes relative to the direction of stress. When a polycrystalline metal is deformed each individual grain tends to orientate itself in a particular manner, so that the wire, rod, or foil, etc., may be considered as having a texture which is definitely related to the direction of the greatest deformation. This was first described by Ettisch, Polanyi, and Weissenberg‡ who called it 'Faserstruktur' in reference to its resemblance to the structure of a large number of natural fibres such as silk, cellulose, muscle and nerve fibres, etc. §

The structure longitudinal to the wire axis is therefore different from that in the cross-section.

The resulting orientation depends on the structure of the metal and the nature of the deformation. Even metals having the same type of lattice show different degrees of preferred orientation. The end position is the same, however, whether one starts with a single crystal or normal polycrystalline metal, except in the case of a tensile test.

4. Experimental determination of orientation in metal wires, foils, etc.

A monochromatic X-ray photograph taken through, or reflected from, a normal polycrystalline metal, shows on the film a number of

† C. A. Edwards, L. B. Pfeil, *J. Iron Steel Inst.* **112** (1925), 79.

‡ M. Ettisch, M. Polanyi, K. Weissenberg, *Z. Physik*, **7** (1921), 181; *Z. phys. Chem.* **99** (1921), 332.

§ Herzog Jancke, *Z. Physik*, **3** (1920), 196.

concentric rings of uniform density due to reflections from the principal spacings of the little crystals which are orientated at random (Fig. 3, Plate IV). If the crystals are orientated so that a particular crystallographic direction lies in the axis of the wire, the structure is no longer isotropic and the rings are broken up into patches of greater and lesser intensity according to the degree of preferred orientation in the wire (Fig. 4, Plate IV). From the angular relationships be-

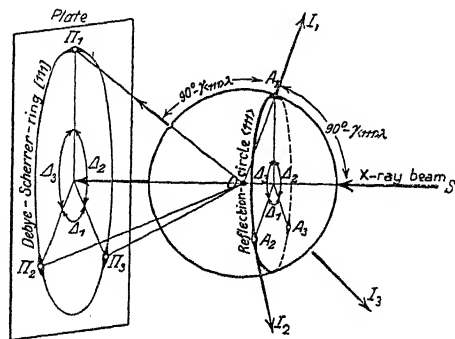


FIG. 30. Diagram showing relation between reflecting planes and spots on photographic plate.

I_1, I_2, I_3 = normals to reflecting planes.

II_1, II_2, II_3 = corresponding spots on photographic plate.

tween the wire axis, the X-ray beam, and the spots, it is possible to calculate the position of the reflecting planes which give rise to them.

The above diagram (Fig. 30), which is due to Mark and Weissenberg,[†] shows the relationship between the X-ray beam, a Debye-Scherrer ring on the plate with spots corresponding to reflections from three groups of octahedral planes, and the positions of these points depicted on a sphere. γ is the reflecting angle for all planes of the spacing of the $\{111\}$ planes and is fixed by the wave-length of the X-rays λ and the nature of the metal, according to the Bragg formula.

The diagram shows that the angles between interference points on the Debye ring of the plate are identical with the angles between the planes or groups of reflecting planes in the specimen. The same methods may be applied when the specimen is stationary or rotated. In the latter case, the spots lie on layer lines.[‡] (Figs 5, 6, Plate IV.)

[†] H. Mark, K. Weissenberg, *Z. Physik*, **14** (1923), 328.

[‡] The theory of the production of 'fibre-structure' diagrams is dealt with in the following papers by Polanyi and Weissenberg, *Z. Physik*, **7** (1921), 149; **9** (1922), 123; **10** (1922), 44.

5. Structure produced by tension

To take the simplest case first; that of a wire. If all the grains are arranged so that one direction always lies parallel to the wire axis, the other planes in all the crystals will also make the same angles with the axis, although arranged at any angle round the axis. As a result of normal tension and also wire-drawing, the direction taken up by the crystals of a body-centred cubic lattice is such that the face diagonal of the cube, i.e. $[110]$, lies parallel to the wire axis. In face-centred cubic crystals the chief direction is $[111]$, but a $[100]$ axis also occurs to a greater or less extent according to the particular metal. This may be described as a 'double fibre-structure'. This orientation is not the same as that taken up by single crystals in the final stage of a tensile test. Here, a $[112]$ direction lies parallel to the axis. The preferred orientation is not exact in relation to the wire axis. The principal crystallographic direction is generally inclined to the wire axis. It was shown by Schmid and Wassermann,[†] however, that this deviation decreased from the periphery to the inside of a wire. They also showed that the preferred orientation was more complete in the inside layers of a wire. This information was obtained experimentally by dissolving off layers of a copper wire after successive photographs. The table gives figures illustrating the change of inclination of the axis of the fibre structure towards the axis of the wire.

$2r$ mm.	γ	$2r$ mm.	γ
1.75	2°	0.9	4°
1.6	9°	0.4	~ 1
1.3	6°		

r = distance of photographed layer from centre of wire.

γ = inclination of $[111]$ direction to wire axis.

Another measurement of the degree of orientation through a wire section was made by Wood.[‡] Successive layers of the wire were dissolved off and X-ray photographs taken. The degree of preferred orientation was determined by measuring the intensity of the Debye-Scherrer rings in different directions photometrically. If there is no preferred orientation, the relative intensity of rings of the (200) and (111) spacings, for example, should be the same in all directions. If the distribution is not uniform the relative intensity of the rings will

[†] E. Schmid, G. Wassermann, *Z. Physik*, **42** (1927), 779.

[‡] W. A. Wood, *Phil. Mag.*, **1** (1931), 610.

vary. The results showed no orientation in the outermost layers of the wire. In a wire 1.76 mm. diam. the 'Faserstruktur' suddenly appeared at a depth of 0.95 mm. and was nearly constant throughout the cross-section. As drawing was carried farther the ratio of the core to the outer layer decreased from approximately 0.5 to 0.41 when the wire was reduced to 0.73 mm. diam. The degree of orientation meanwhile increased in the core.

Schmid and Wassermann† have calculated the kind of rotation diagram that should be obtained assuming that both a [111] and a [100] axis are parallel to the wire axis in face-centred cubic metals. They have compared the actual photographs obtained from copper, aluminium, silver, and gold, and from intensity measurements have determined the proportional amount of the two structures. These are very different in the four metals as the following table shows:

Metal	Percentage of crystals with	
	[100] parallel to the wire axis	[111] parallel to the wire axis
Aluminium	0	100
Copper	40	60
Gold	50	50
Silver	75	25

6. Compression structures

Compression of a polycrystalline metal produces a fibre structure in the same way as drawing or rolling. For face-centred cubic metals‡ the direction perpendicular to the plane of compression is [110]. This is also the final position taken up by the axes of single crystals of aluminium under compression.§ Polycrystalline specimens and single crystals do not behave similarly in tension. Ono also considers that a [100] direction lies in a radial position.

In body-centred cubic crystals such as iron, both [111] and [100] directions lie in the direction of compression and a [110] direction radial. This is only possible in the case of those crystals in which a [100] direction lies in the direction of compression and most of the evidence is in favour of [111] only.

† E. Schmid, G. Wassermann, *Z. Physik*, **42** (1927), 779.

‡ A. Ono, *Mem. Coll. Engin. Kyushu Imp. Univ.* **3** (1925), 195. G. Sachs and E. Schiebold, *Z. Verein. deuts. Ing.* **69** (1925), 1559. F. Wever, *Z. techn. Physik*, **8** (1927), 404. W. G. Burgers, *Z. Physik*, **67** (1930), 605.

§ G. I. Taylor, *Proc. Roy. Soc. A*, **116** (1927), 16.

Sachs and Schiebold† prepared cubes cut from polycrystalline aluminium and also single crystals and compressed the cube in three directions in turn. In this way the metal still retained its shape after compression. The resulting structure could be interpreted as consisting of three $[110]$ directions perpendicular to three faces of the cube, i.e. parallel to the three directions of compression.

The effect of alternating tension and compression is to produce a structure resembling drawn wires, i.e. a $[111]$ in the direction of tension or compression.

In magnesium‡ the hexagonal axis appears to be parallel to the direction of compression.

7. Structure of rolled metal

1. *Metals of face-centred cubic crystal structure.* The first investigation of the structure of rolled metal foils was carried out by Mark and Weissenberg.§ For the face-centred cubic metals aluminium, copper, platinum, silver, and gold, the following results were obtained. The direction approximately parallel with the direction of rolling was $[11\bar{2}]$. The angle between direction of rolling and the $[11\bar{2}]$ direction was 8° . The direction at right angles to the direction of rolling was parallel with the $[111]$ direction and those directions which lie in the $(11\bar{2})$ plane and make an angle of less than 35° with the $[111]$ direction. Parallel with the plane of the foil there is $[1\bar{1}0]$ and those directions which lie in the $(11\bar{2})$ plane and make an angle of less than 35° with the $[110]$ direction. This was in agreement with Uspenski and Konobejewski,|| but not entirely with subsequent investigations.

In the case of aluminium, Wever†† and Owen and Preston‡‡ found a $[111]$ direction parallel with the rolling direction and a $\{112\}$ plane parallel to the plane of rolling. The different results obtained by different investigators may be attributed partly to variations in the manufacture of the foils, but still later experiments indicate a double 'Faserstruktur' as in the case of drawn wires. In aluminium, copper, and silver foils§§ the ratio of the two directions $[112]$ and $[111]$ was

† G. Sachs and E. Schiebold, *Z. Verein. deuts. Ing.* **69** (1925), 1559.

‡ S. Tsuboi, *Mem. Sci. Eng. Kyoto Univ.* **11** (1928), 375.

§ H. Mark, K. Weissenberg, *Z. Physik*, **14** (1923), 328; **16** (1923), 314.

|| N. Uspenski, S. T. Konobejewski, *Z. Physik*, **16** (1923), 215.

†† F. Wever, *Z. Physik*, **28** (1924), 69. F. Wever, W. Schmidt, *Z. techn. Physik*, **43** (1927), 741.

‡‡ E. A. Owen, G. D. Preston, *Proc. Phys. Soc. London*, **38** (1926), 132.

§§ E. Schmid, G. Wessermann, *Z. Physik*, **42** (1927), 779.

found to be 1.4 : 1. The results generally indicate a double structure with $[112]$ in the rolling direction and $\{110\}$ in the plane of rolling, and $[111]$ in the direction of rolling and $\{112\}$ in the plane of rolling.

As in the case of drawn wires, either or both of these structures may be present to varying degrees. A number of alloys† have also been investigated in addition to the metals already mentioned and a summary of results will be found in the following table:

Metal	Type of Lattice	Direction parallel to direction of rolling	Crystal plane in plane of rolling
Silver	Face-centred cubic	$[112]$	$\{110\}$
Brass			
Platinum			
Gold-silver 30% Ag			
Copper-tin			
Aluminium		1. $[112]$ 2. $[111]$	1. $\{110\}$ 2. $\{112\}$
Copper			
Nickel			
Constantan			
Gold		Slightly different	
Gold-silver 5% Ag			

In order to depict these structures in some simple form Wever‡ constructed stereographic diagrams (Pole figures) in which areas in the figures corresponded to the positions of the octahedral, dodecahedral, etc. planes relative to the direction of rolling, etc. By means of shading, the degree of preferred orientation can be represented. These figures show very clearly the distribution of a particular set of planes relative to the direction and plane of rolling, and also in what positions there is the greatest concentration of reflecting planes of a particular type. Examples are shown in Figs. 31 and 32.

2. *Metals of cubic body-centred crystal structure.* There seems to be general agreement about the structure of foils of body-centred cubic metals.‡ In iron and molybdenum a face diagonal $[110]$ direction lies in the direction of rolling and a cube $\{100\}$ plane in the plane of rolling. Two other positions have been found,§ however, one in which $[110]$ is in the direction of rolling and $\{112\}$ parallel to the plane of rolling; and the other $[112]$ in direction of rolling and $\{111\}$ in the plane of rolling.

† von Göler, G. Sachs, *Z. Physik*, **56** (1929), 477.

‡ F. Wever, *Z. Physik*, **28** (1924), 69.

§ G. Kurdjumow, G. Sachs, *Z. Physik*, **62** (1930), 592.

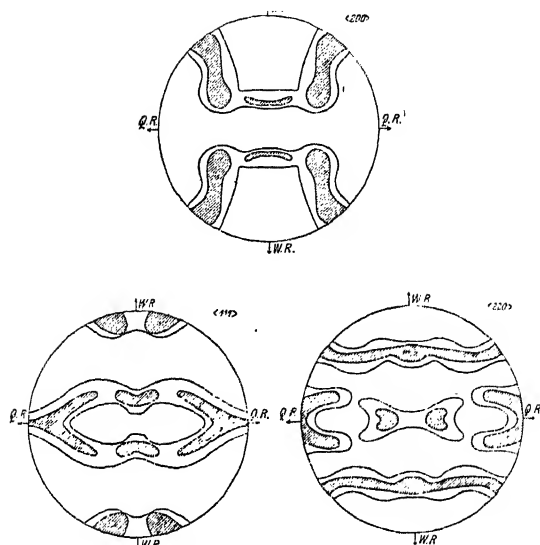


FIG. 31.

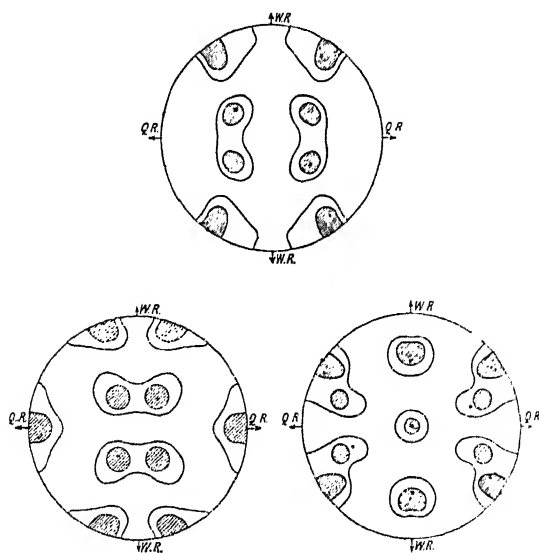


FIG. 32.

FIGS. 31 and 32. Pole figures of cube, octahedral, and dodecahedral planes. 31. Aluminium sheet. 32. Brass sheet.

W.R. = direction of rolling.

Q.R. = direction at right angles to direction of rolling.

3. *Metals of hexagonal crystal structure.* The structure produced by rolling zinc into sheet has now been established by Schmid and Wassermann.† The hexagonal axis lies in the plane containing the direction of rolling and the normal to the plane of rolling and at an angle of 20° to the latter. The basal planes lie chiefly in the direction of rolling and are entirely absent in the direction at right angles to this plane (Fig. 33).

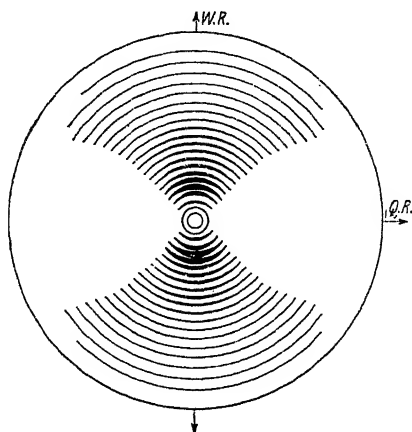


FIG. 33. Rolling structure of zinc.

The prism planes of the first order are arranged accordingly, i.e. their normals make angles of from 90° – 60° with the plane normal and have no preferred directions in the plane of rolling. A previous investigation of the structure of rolled zinc and cadmium by Mark‡ placed the hexagonal axis at 50° to the direction of rolling and 70° to the foil normal in the plane at right angles to this direction.

Still another result was obtained by Davey, Nitchie and Fuller.§ According to them, the structure could be regarded as a 'Spiralfaser' of which the axis coincided with the direction of rolling. The hexagonal axis and the direction of rolling made an angle of $61 \pm 18^\circ$.

These differences must be attributed to different methods of rolling in the preparation of the sheets or foils. It is quite clear that, if the sheet was cross-rolled at any stage in its manufacture, different results might be obtained.

† E. Schmid, G. Wassermann, *Metallwirtschaft*, 9 (1930), 689; *Z. Metallkunde*, 23 (1931), 87.

‡ H. Mark, *Z. Krist.* 61 (1925), 75.

§ W. P. Davey, C. C. Nitchie, M. L. Fuller, *Amer. Inst. Min. Met. Eng. Techn. Publ.*, No. 243, 1929.

Schmid and Wassermann[†] also compared the structure on the surface of a sheet with that at the centre and found differences comparable with those in wires. On the extreme surface the basal plane was nearly parallel with the plane of rolling. At a depth of 0.025 mm. it had reached the 20° position described above.

In the case of magnesium,[†] the hexagonal axis coincided with the sheet normal and the basal planes lay in the plane of rolling. The

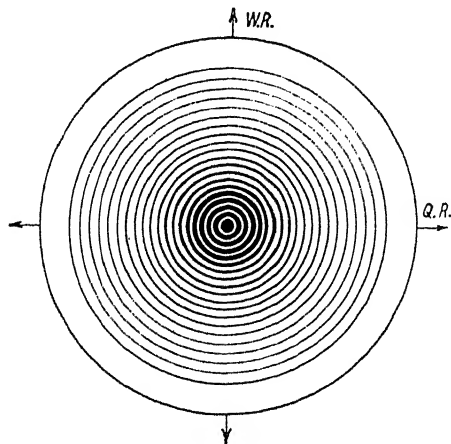


FIG. 34. Rolling structure of magnesium.

distribution about this direction was almost uniform, i.e. there was very little difference between the direction of rolling and that at right angles. (Fig. 34.)

Both these metals have two methods by which they may deform, i.e. by translation on the basal plane and by twin formation on the pyramidal, second order, planes. (See section on deformation by twinning.) According to the orientation of the crystal, twinning may result either in an increase or decrease in length of the specimen. When the position of the axis lies between 0° and 50° to the hexagonal axis there is a shortening on twinning on any of the six possible twin planes. For positions which are about 90° to the hexagonal axis the result is an increase in length. Intermediate directions result in either extension or contraction. Fig. 35 is a stereographic projection of the zinc crystal by means of which this may be explained. According to the position of the point representing the longitudinal axis of the

[†] Schmid and Wassermann, loc. cit.

specimen in the triangles A, B, C, or D, twinning on the six possible twin planes $\{10\bar{1}2\}$, I to VI in the diagram, may lead to extension or contraction as follows:

- A. Twin planes I to VI result in contraction.
- B. Twin planes II, III, V, and VI, contraction.
Twin planes I and IV, extension.

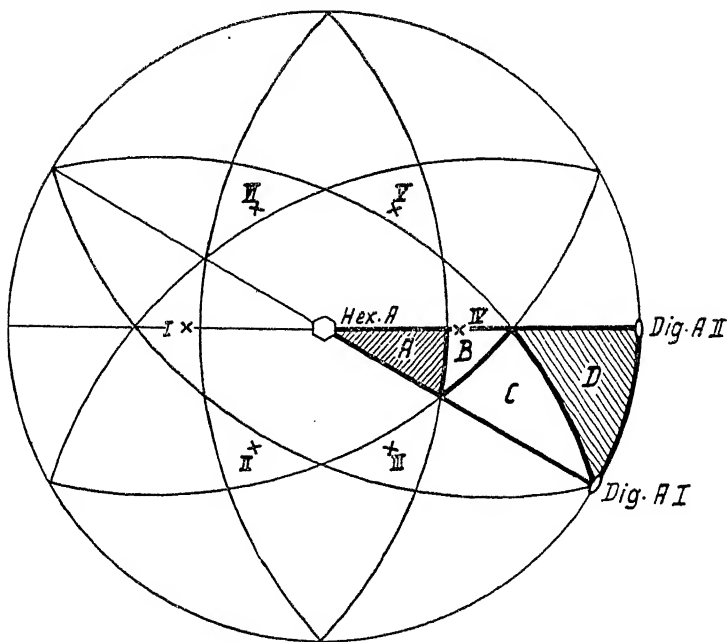


FIG. 35.

- C. Twin planes II and V, contraction.
Twin planes I, III, IV, and VI, extension.
- D. Twin planes I to VI, all extension.

As the axial ratio of magnesium is 1.62, compared with zinc 1.86, these directions are reversed, and a form of twinning which leads to an extension in zinc leads to contraction in magnesium.

From these considerations, it is possible to explain the differences in the rolling structures of these two metals. The greatest amount of deformation takes place by means of slip on the basal plane. This brings the basal plane nearer and nearer to a position parallel to the plane of rolling and the hexagonal axis perpendicular to it. In the case of magnesium, twinning cannot assist the deformation, as it

would lead to a contraction of the sheet in the direction of rolling (or drawing). The process is the same to begin with in zinc, but here, twinning results in an increase in length of the specimen. Moreover, after twinning, the basal plane in the new position is favourably situated for further extension, i.e. it is about 60° to the direction of extension. The absence of basal planes situated at right angles to the direction of rolling and normal to the plane of rolling can be explained, if one considers that slipping in crystals so orientated cannot lead to any increase in length of the specimen and deformation must be by the formation of twins only. By twinning, their orientation is altered to one more favourable to deformation by slipping.†

8. Torsion structures

Ono‡ has studied the structure of wires which have been subjected to torsion and, in the case of aluminium and copper, finds a [111] direction parallel to the longitudinal axis of the wire and a [110] direction tangential to it. Sachs and Schiebold§ find a double fibre-structure, i.e. both [111] and [100] directions parallel to the wire axis. For iron, Ono‡ finds both [110] and [112] parallel to the wire axis and [111] tangential. These directions are almost identical with similar directions in drawn wires and it is open to doubt whether they represent true structures produced by torsion, or are due to the fibre-structure produced by drawing the original wires. Owing to the nature of the deformation, the experimental difficulties of determining the structure are great.

9. Explanations of the formation of fibre-structures

There is considerable resemblance between rolling and compression as a method of deformation and it has been shown that if a metal is compressed so that it cannot flow in all directions but only in one direction, the structure resembles that of rolled metal.||

Several attempts have also been made to explain how the fibre-structure in foils can be produced by reorientation of the crystals in a manner similar to that which occurs during extension and compres-

† For a fuller account of this work, see Schmid and Wassermann, *loc. cit.*, and the same authors, *Handbuch der physikalischen Mechanik*, Band iv, *Technische Physik der festen Körper*.

‡ A. Ono, *Mem. Coll. Engin. Kyushu Imp. Univ.* **3** (1925), 195.

§ G. Sachs and F. Schiebold, *Naturwiss.* **13** (1925), 964.

|| S. Tsuboi, *Mem. Sci. Eng. Kyoto Univ.* **11** (1928), 375. F. Wever and W. E. Schmidt, *Mitt. K. W. Inst. Eisenf.* **11** (1929), 109.

sion of a single crystal. It most nearly resembles the structure produced by compression in that, in a face-centred cubic metal, a $[110]$ direction is parallel (1) to the plane normal of a sheet; (2) to the direction of compression in a crystalline aggregate; (3) to the normal of a compression disk from a single crystal in its final position.

Let us consider further the result of compression on a section of an aluminium crystal,[†] when the crystal axes reach a stable position, with a (011) plane perpendicular to the direction of compression, i.e. in the face of the specimen. The two slip-planes (111) and $(\bar{1}11)$ are symmetrically placed, their line of intersection being a direction of indices $[01\bar{1}]$ at right angles to the face normal, while the direction at right angles to both of these is $[100]$.

In this position, further compression reduces the thickness parallel to $[011]$, does not change the dimension parallel to $[01\bar{1}]$, and leads to an increase in the dimension parallel to $[100]$. In each octahedral plane there are two possible directions of slip (in this symmetrical position), i.e. on (111) slip may be towards the pole of the $(10\bar{1})$ or $(1\bar{1}0)$ plane, on $(\bar{1}11)$ slip may be towards the pole of the (101) or (110) plane. If slip occurred in one direction only on each plane, there would be a displacement on further distortion of one surface of the specimen relative to the other. But if slip occurred equally in both directions on each plane, then the distortion of the compressed disk would be identical with that obtained by rolling, i.e. extension in one direction; no change in dimension at right angles to the direction of rolling; and a reduction in thickness. The crystal structure obtained by this type of deformation, however, is not that which is found for the structure of foils by means of X-rays.

There is another possibility. The stable position for crystal axes in a tensile test of aluminium is reached when the axis of the specimen coincides with a $[112]$ direction. Further distortion leads to a reduction in one dimension in a direction of type $[110]$ whereas the $[111]$ direction perpendicular to $[110]$ and $[112]$ remains constant. Masing and Polanyi[‡] suggested that such a distortion could also account for the structure of rolled foils, etc. The $[112]$ direction would be in the direction of rolling; the $[110]$ direction normal to the plane of rolling; and the $[111]$ direction at right angles to the direction of rolling. This type of deformation would give constant breadth of the sheet

[†] G. I. Taylor, *Proc. Roy. Soc. A*, **116** (1927), 16.

[‡] M. Polanyi, G. Masing, *Ergebn. exakt. Naturwiss.* **2** (1923), 177.

and decrease in thickness, but as Taylor has pointed out, lines of particles in the plane of rolling at right angles to the direction of rolling do not remain so, and the effect of rolling would be to change a rectangular specimen into a rhombohedral one.

In a more general manner Polanyi† has attempted to correlate the structure found in wires and foils with the distortion of the individual crystals. This is necessarily extremely difficult because one cannot estimate the influence which one crystal exerts on another although it is known to exist. (See p. 52.) It may be quite sufficient to affect both choice of slip-plane and direction of slip. As the individual grains become elongated the nature of their contact is altered, so that this factor is a constantly changing one and almost impossible to estimate.

At the present moment there is no satisfactory explanation of these structures. From the work on single crystals it is to be expected that the individual grains will take up some more or less definite position relative to the direction of the applied stresses. But the conditions are so entirely different that if each grain behaved as it would in an isolated state, there would be no cohesion possible between them, and the metal would fall in pieces. In order that the crystals shall keep in contact, slip must take place on more than one plane and in more than one direction, and it is impossible to predict what the final orientation is likely to be. When, by reason of symmetry, only one slip-plane is available, e.g. in zinc, the metal resorts to twinning in order to adapt itself to change of shape.

It has also been shown by many investigators that a similar end structure is obtained, whether one starts with a single crystal or an aggregate.‡ This suggests that the original orientation has nothing to do with the formation of a fibre-structure. On the other hand, there must be some reason for the orientations that are found, and a connexion between them and the ability of a metal to deform.

Körber§ explains the formation of a fibre-structure, on the grounds that the crystals orientate themselves symmetrically in relation to the principal stresses so that they present planes and directions most able to resist further deformation. In his opinion this would account for

† M. Polanyi, *Z. Physik*, **17** (1923), 42.

‡ S. Tanaka, *Mem. Coll. Science. Kyoto Imp. Univ. A*, **10** (1927), 303, finds some influence of the original orientation but this depends largely on the degree of deformation. See also W. Burgers, chap. 14.

§ F. Körber, *Stahl und Eisen*, **48** (1928), 1433.

the increase in strength due to working, but there is much evidence that orientation alone cannot account for it. Körber's† explanation is based on the different properties sometimes exhibited in test-pieces cut at right angles to and parallel with the direction of rolling.

The part played by orientation alone in the problem of hardening, etc., has already been demonstrated in the case of single-crystal test-pieces. In polycrystalline metal the maximum effect is obtained with crystals of hexagonal symmetry which have only one available slip-plane, and in cold rolled zinc‡ sheet the strength is as much as 25 per cent. higher in directions at right angles to the rolling direction than parallel with this direction. In pure aluminium, however, it is possible to anneal a hard-drawn wire so that it retains the same fibre-structure even after complete mechanical softening and recrystallization have taken place.§ The values for the hard-drawn wire are: tensile strength 20.6 kg./mm.²; extension 1 per cent.; while the corresponding values for the annealed wire having the same orientation are 3.5 kg./mm.² and 5 per cent. respectively. A similar result has been obtained with an aluminium-silicon eutectic alloy in which X-ray photographs showed no difference in structure.

10. Stress-strain relationships

The first effect of a stress applied to polycrystalline metal is an elastic distortion. The load required to reach the elastic limit is very small for soft metals such as annealed copper. Within this range the distortion is not permanent. When the load is increased beyond this point plastic deformation sets in. The actual load necessary to produce the first minute permanent extension is very difficult to determine experimentally. Except in the case of certain steels, in which the beginning of plastic deformation is accompanied by a sudden, considerable yielding of the metal, the change-over from one to the other is gradual. The yield-point is generally defined for practical purposes as the load required to produce a permanent extension of 0.2 per cent.

A typical stress-strain curve is shown in Fig. 36.|| p is the limit of proportionality or elastic limit. σ is the yield-point. During the earlier

† F. Körber, *Z. Elektrochem.* **29** (1923), 295.

‡ E. Schmid, G. Wassermann, *Z. Metallkunde*, **23** (1931), 87.

§ E. Schmid, G. Wassermann, *Z. techn. Physik*, **9** (1928), 106. von Göler, G. Sachs, *Z. Physik*, **56** (1929), 495.

|| G. Masing, N. Polanyi, *Ergebn. exakt. Naturwiss.* **2** (1923), 177.

stages of deformation the rate of increase of load required to produce a given extension is greater than during later stages. This is also the case in single crystals. As soon as the yield-point is reached plastic deformation begins, with a corresponding decrease in the cross-sectional area of the test-piece. A continually decreasing cross-sectional area is thus supporting a greater and greater load. If the stress is plotted against reduction of area of the cross-section or if it is calculated on the actual cross-section at each stage of the test and

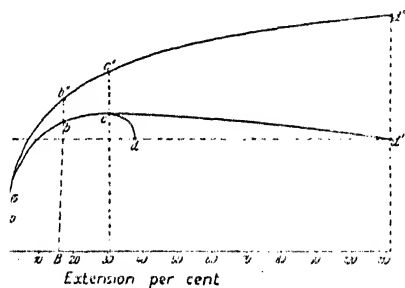


FIG. 36.

plotted against the extension, a truer picture of the stress-strain relationship is obtained. The lower curve of Fig. 36 gives the load-extension relationship, the upper the stress-extension. If, under these conditions, the curve still continues to rise, it is proof that the resistance to shear increases with the deformation. During this part of the extension, abc and $ab'c'$, the distortion is uniform. A stage is finally reached when there is a local contraction and a sudden drawing down, c and c' , of the metal to fracture d and d' . The extension between the yield-point and the beginning of fracture is a measure of the ductility of the metal. It also determines, within limits, what the final breaking load will be, as the greater the extension, the more the work-hardening. This is very marked in certain examples of single crystals (see Fig. 19).

A similar sequence of events occurs in compression tests, with the exception of the final drawing down before fracture. It has been shown that if the amount of deformation is plotted against stress in the case of a tensile, compression, and torsion test of the same metal, the curves follow each other very closely† (Fig. 37). The slight

† P. Ludwik, R. Scheu, *Stahl und Eisen*, **43** (1925), 999. G. I. Taylor, H. Quinney, *Proc. Roy. Soc. A*, **143** (1934), 307. For further discussion on the comparison of the three methods of testing, see G. Sachs, *Z. Verein. deuts. Ing.* **72** (1928), 724.

variations found may be due to wrong assumptions in calculating the shear stress in torsion specimens. This figure should be compared with Fig. 88, Chapter XIII.

It is to be expected that other forms of deformation also have a hardening effect on metals according to the amount of the deformation. It is much more difficult to estimate the work done during the processes of rolling and drawing, and it is only possible to compare

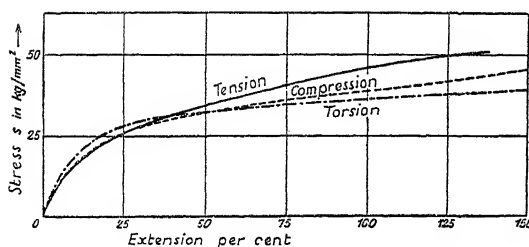


FIG. 37. Stress-strain curves for copper, in tension, compression, and torsion.

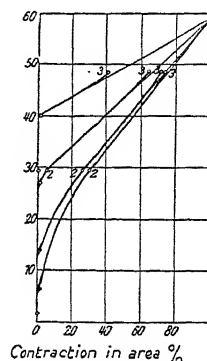


FIG. 38. Series A.

the mechanical properties, after different extensions or reductions in area, in order to obtain some idea of the hardening effects produced.

The following table shows the raising of the elastic limit by cold working:†

Elastic limits

	Soft kg./cm. ²	After compression kg./cm. ²
Cu	203	2,780
Al	283	600
Pb	25	102
Fe	2,370	5,840–6,000
Ni	788	5,570

The yield-point is consequently raised and the amount of extension before fracture reduced. This is to be expected, as a considerable amount of extension has already taken place during deformation by rolling or drawing, etc. Fig. 38 shows the stress plotted against contraction in area for four copper wires which have been drawn down to different amounts before testing. The wires begin to extend at

† G. Masing, N. Polanyi, *Ergebn. exakt. Naturwiss.* 2 (1923), 177.

different values, but according to the investigators necking begins at the same point in each wire, at points 2, and fracture at points 3, also at the same corresponding stress. If the curves are produced, they meet at point 4, corresponding to 100 per cent. reduction in area. v. Möllendorf and Czochralski† showed that the stress at this point, i.e. point 4, was double that at which necking began (2). Point 2 may be said to correspond to the technical breaking stress of a metal. Fig. 38 would indicate that, calculated on this basis, the true breaking stress of a metal is constant whatever the deformation it has been subjected to.

Partial confirmation of this hypothesis is provided by comparison of the breaking stresses of a number of copper wires which have been drawn through dies of different angles to different extensions.‡ If these are compared with a tensile stress-strain curve for the same material in the annealed state, the final breaking stresses for all the specimens tend to approach the same value. But this is not true of the wires of intermediate extensions.

The increase in tensile strength and reduction of elongation produced by progressive amounts of cold-rolling and cold-drawing have been determined by a number of workers on different metals. The results obtained by Alkins§ and Johnson|| for copper, in the case of wire-drawing and rolling respectively, are typical examples. The change is not entirely uniform. Alkins found distinct breaks in the curves of all the properties investigated (Fig. 39). Explanations may be found in the structure of wires and sheets as shown by X-rays, and particularly in the differences existing between the inside and outside layers. It is obvious that the smaller sections will be more susceptible to influences of surface structure. This problem is referred to more fully in the section dealing with the internal structure of wires and foils, but it may be well to point out here that the metal continues to increase in strength after a fibre-structure has been produced, and that consequently the actual increase in hardness cannot be attributed to the change of orientation resulting from its formation. This is comparable with similar phenomena in the extension of single crystals. The increase in resistance to shear due to orientation

† v. Möllendorf, J. Czochralski, *Z. Verein. deuts. Ing.* **54** (1913), 931. See also F. Körber, *Mitt. K. W. Inst. Eisenf.* **3** (1922), 1.

‡ W. Linicus, *Z. Metallkunde*, **23** (1931), 205.

§ W. E. Alkins, *J. Inst. Metals*, **23** (1920), 381.

|| F. Johnson, *J. Inst. Metals*, **13** (1920), 443.

changes is small compared with that which can be attributed to the effects of deformation.

Other forms of testing show a similar increased resistance to deformation. Some results are given in Fig. 39, together with those of tensile tests. The only form of testing which does not exhibit any difference is the scratch hardness test.

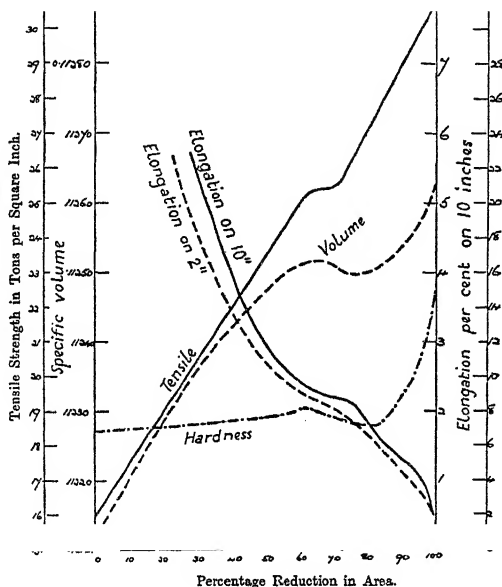


FIG. 39.

Heating the metal to a sufficiently high temperature causes it to recrystallize and all the effects of deformation are removed. (See section on recrystallization and crystal growth.) The metal in the work-hardened state is, therefore, also in an unstable condition. It has been shown in Chapter III that single crystals can acquire all the mechanical properties of the aggregate, so that the explanation of these phenomena must be sought in the changes which take place in the crystal structure.

The effects of mechanical deformation on the structure and mechanical properties of polycrystalline metal may be summarized as follows:

1. The individual grains become distorted.
2. The deformation takes place by gliding on the crystal planes.

3. The presence of crystal boundaries strengthens the material.
4. The crystals tend to take up certain definite orientations in relation to the deformation.
5. The type of crystal lattice determines the orientation taken up.
6. The mechanical properties are changed by plastic deformation.
7. There is a real hardening of the material, i.e. deformation tends to increase the resistance to further deformation.
8. Change in orientation can only account for part of the effects of mechanical deformation.

VI

DEFORMATION OF ALLOYS

IN considering the distortion of alloys it is necessary to distinguish between single-phase alloys, and those in which two or more constituents are present. To the first category belong solid solutions; that is to say, the metals are mutually soluble in the solid state and cannot be distinguished under the microscope. They most closely resemble the pure metals in both their physical and mechanical properties. The extent to which the crystal structure is altered by the presence of another metal in solution varies from one case to another. The lattice may be expanded or contracted according to the alloying element. The available evidence is in favour of a uniform distortion by replacement of atoms in the lattice without any alteration in the type of lattice; only in the dimensions. It is not surprising to find that the deformation of solid solution alloys closely resembles that of the predominating pure metal, although the forces required to produce the deformation are usually greater.

On the other hand, where two or more phases are present, the deformation of both phases must be considered and also their influence on each other.

Single crystals. 1. Zinc alloys

One of the earliest investigations of alloy crystals was that of Rosbaud and Schmid† on zinc crystals containing small quantities of cadmium and tin. The crystals were prepared by the Czochralski-Gomperz method. They were found to be truly uni-crystalline. Ductile crystals could be produced up to 1.03 per cent. cadmium, and 2 per cent. tin. Only traces of the eutectic were visible with 1.03 per cent. cadmium, but in the case of tin the eutectic formed with a concentration of less than 0.10 per cent. The two alloys therefore represent examples of different types of solution. In the former, the cadmium atoms enter the zinc lattice; in the latter, there is no solution in the zinc, but a second constituent separates on the basal planes of the zinc lattice. In both cases deformation takes place by means of slip on the basal plane as in zinc. The difference in properties is clearly brought out when the shear stress on the slip plane at the yield-point is plotted

† P. Rosbaud, E. Schmid, *Z. Physik*, **32** (1925), 197.

against cadmium and tin content respectively. The curves are shown in Fig. 40. The increase in shear stress is very much greater for cadmium than for the same amount of tin. In other words, the soluble constituent influences the mechanical properties to a greater extent than the insoluble.

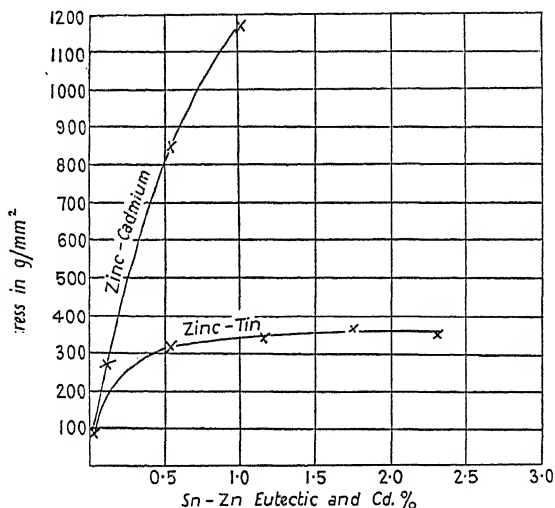


FIG. 40. The effect of additions of cadmium and tin on the shear stress at the yield-point of zinc crystals.

2. Aluminium-zinc alloys

The influence of increasing the percentage of a second metal which is completely soluble in the first is shown in the change of properties of aluminium-zinc alloys.† The limit of solubility of zinc in aluminium is approximately 18.5 per cent. The crystals were made by the method of straining and heat treating from extruded bars of the required composition. Distortion and X-ray measurements were made of crystals containing 10 and 15 per cent. zinc. These deformed like aluminium by means of slip on the octahedral plane and in a direction normal to a {110} plane. The shear stress on the slip-plane is plotted against amount of shear, for various alloy crystals in Fig. 41. The presence of zinc increases the resistance to shear. The actual figures for elongation and maximum stress varied considerably from crystal to crystal, but the general result of alloying is to lower the elongation figures and increase the figures for maximum stress. The

† C. F. Elam, *Proc. Roy. Soc. A*, **109** (1925), 143; *A*, **115** (1927), 133.

alloy crystals start harder and do not distort to such an extent before fracture. This is in agreement with the appearance of the crystals, and their fracture (Figs. 1, 2, and 3, Plate III). Round bars of alloys

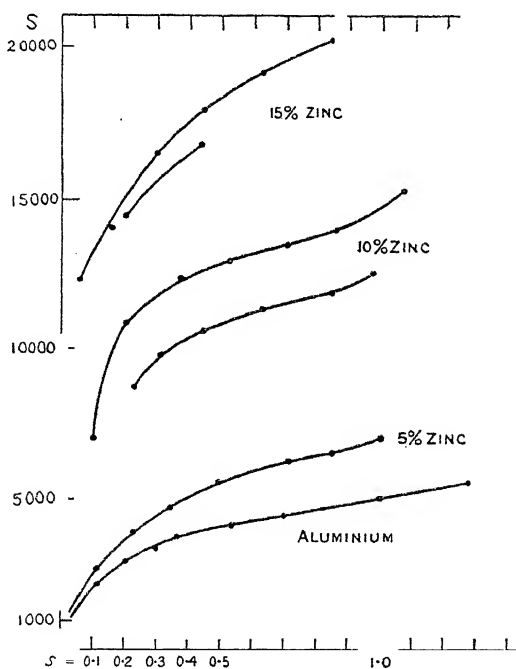


FIG. 41.

S = shearing stress measured in pounds per square inch.
 s = amount of shear.

of 5 and 7.5 per cent. zinc become elliptical in cross-section and the fracture is usually wedge-shaped. Alloys of 10 and 15 per cent. also become elliptical in cross-section, but elongate to a less and less extent. The fracture consists of a single plane at 45° to the axis and is usually parallel to the slip-plane (Fig. 2, Plate III). As these crystals fracture before double slipping begins, unless the initial position is such that two octahedral planes are equally inclined to the axis, the type of fracture is not unexpected and may be compared with that of metals like zinc, cadmium, magnesium, etc. With 18 per cent. zinc the extension before fracture is even less and the plane of fracture is either parallel with the slip-plane, i.e. a $\{111\}$ plane, or perpendicular to it, a $\{112\}$ plane. In one example only,

slip-planes corresponding with $\{112\}$ planes appeared and the fracture consisted of two planes, which were also of this type. The fracture, extension and breaking loads depend on the original orientation of the crystal relative to the applied load.

3. Copper-zinc and copper-aluminium alloys

Copper can hold 38 per cent. zinc in solid solution. Large α brass crystals can be made by lowering a graphite tube containing a rod of metal through a furnace so that the metal freezes from one end (Bridgman's Method). The crystals are not homogeneous, but most of the coring is removed by annealing. Distortion and X-ray measurements† show that the crystals deform like pure copper. When, however, they reach the point when slipping on two octahedral planes would be expected, this does not occur. Slip proceeds still further on the first plane and then suddenly changes over on to the second octahedral plane. Double slipping actually never takes place—either one or other plane functioning at a time.

This can be proved by distortion measurements and by studying the movement of the test-piece axis in relation to the crystal axes. If, in the calculation of the distortion, reference is always made to the measurements before testing, it will be found that, up to a certain point, slip is confined to one plane, i.e. a $\{111\}$ plane. After that, the undistorted cone changes its form and agrees with no crystal plane. If, however, the distortion from one small extension to the next is determined, the change-over from one octahedral plane to another is unmistakable. This is shown in Fig. 43. All the measurements are in agreement. The movement of the test-piece axis is illustrated in Fig. 42.

In this particular crystal three separate stages in the deformation could be identified, Nos. 1 and 3 being on the same octahedral plane. From $\epsilon = 1.91$ to $\epsilon = 2.59$ the direction of slip as shown in Fig. 42 is not accurately towards the $[110]$ direction, but from distortion measurements the agreement is much closer. It is important to note that in distortion of this kind, the measurement of slip-bands may be misleading, as old markings persist on the surface even when another set represents the true slip-plane. In order to avoid any error that might arise in this way, it would be necessary to repolish the surface at intervals.

† C. F. Elam, *Proc. Roy. Soc. A*, **115** (1927), 148.

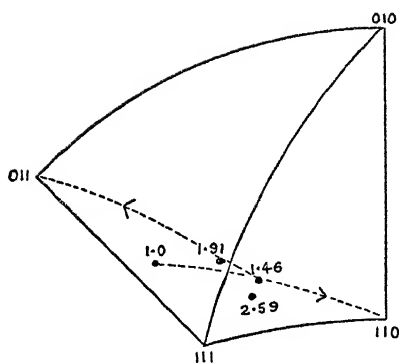
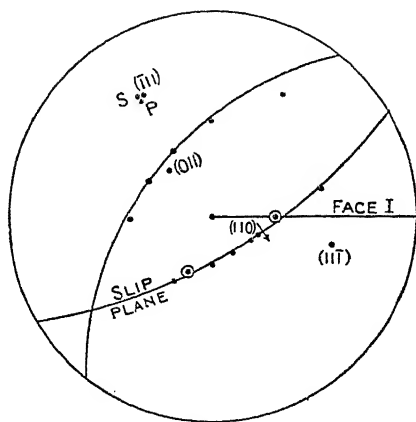
(a) α Brass.

FIG. 42.

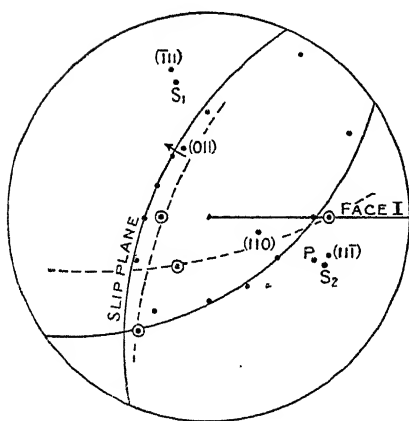
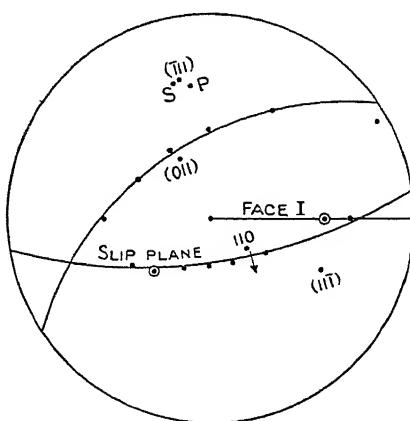
(b) α Brass. $\epsilon = 1.0$ to 1.46 . Strained position. \odot = traces of slip-bands. P = pole of slip-plane from distortion measurements. S = pole of slip-plane from slip-bands.(a) α Brass. $\epsilon = 1.465$ to $\epsilon = 1.915$. Strained position. \odot = traces of slip-bands. P = pole of slip-plane from distortion measurements. S = pole of slip-plane from slip-bands.

FIG. 43.

(b) α Brass. $\epsilon = 1.915$ to $\epsilon = 2.595$. Strained position.

The behaviour of brass crystals was confirmed and amplified by Sachs and his collaborators.[†] The higher the zinc content, the greater the tendency to go on slipping longer on one plane than would be

[†] G. Sachs, H. Shoji, *Z. Physik*, **45** (1927), 776. G. Sachs, M. Masima, *ibid.*, **50** (1928), 161; **51** (1928), 321. von Güler, G. Sachs, *ibid.*, **55** (1929), 581.

expected. It is more surprising when one considers the distribution of shear stress on the two planes. If the value of the shear stress on each plane is plotted against extension, the two curves cut at points which represent the crossing of the boundary from one spherical triangle to the other in Fig. 42. In copper or aluminium double slip would begin where the two curves first intersect. The first plane

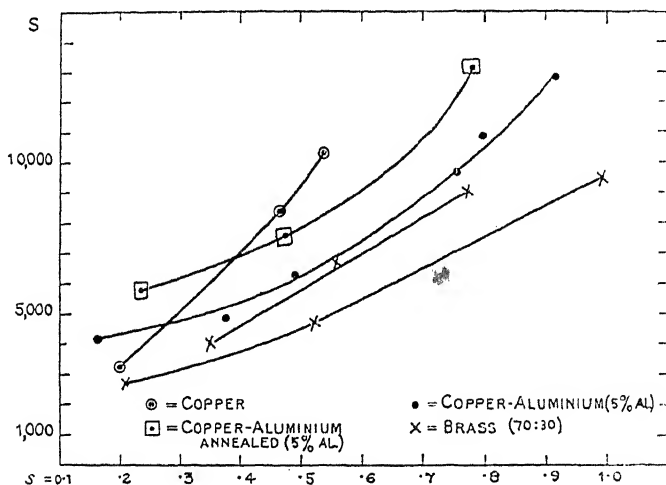


FIG. 44.

continues to function, however, although the shear stress on the second plane is much greater. It is difficult to say how much the change-over from one plane to another is influenced by stopping the test for the purposes of measurement, etc. A test carried right through without stop, still showed an abrupt change of slip-plane at certain stages of the test. A new set of slip-planes was seen to appear suddenly at one point in the crystal and gradually spread. Fig. 4, Plate I is a photograph of such a crystal in which the load was removed before the change was complete.

Tests on a copper alloy† containing 5 per cent. aluminium gave similar results (Fig. 3, Plate I). Here again the crystals deformed by slip on one plane longer than would be expected from symmetry considerations and then changed over suddenly to the second octahedral plane. This was confirmed by distortion and X-ray measurements.

One of the most remarkable characteristics of these crystals is their

† C. F. Elam, *Proc. Roy. Soc. A*, **116** (1927), 694.

ductility. A copper-zinc alloy extended 168 per cent. The highest value for a copper-aluminium alloy crystal was 138 per cent.

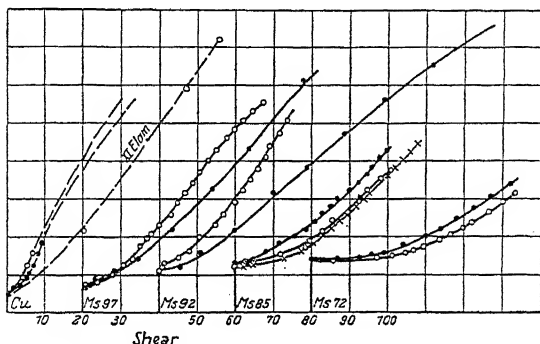


FIG. 45. Tensile tests of copper-zinc alloy crystals.
Nos. refer to percentage copper.

The shear-stress on the slip-plane plotted against amount of shear for some of these crystals and pure copper may be compared in Fig. 44. This may be compared further with Fig. 45, which gives the results of a number of brass crystals of different zinc content. There is considerable variation from crystal to crystal, but it is quite clear that the increase in resistance to shear with deformation is less for the alloys and decreases as the zinc content increases. On the other hand the yield-point is raised as Figs. 45 and 46 show. It appears that the alloying atoms assist deformation rather than hinder it, although the alloys are actually harder if judged by their yield-points and breaking loads. The distortion of β brass crystals has been fully described in Chapters II and III.

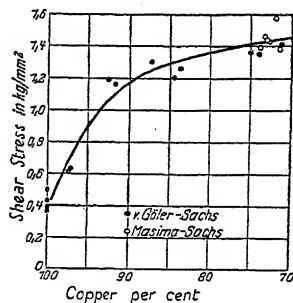


FIG. 46. Copper-zinc alloy crystals.

4. Copper-gold alloys

Alloys with a composition corresponding to the compound AuCu_3 undergo a structural change at about 380°C . Below this temperature the lattice is tetragonal face-centred, with an axial ratio of 1.069–1.08, and above, it has a simple face-centred cubic structure.† Sachs and

† W. Gorsky, *Z. Physik*, 50 (1928), 64.

Weerts† carried out mechanical tests on crystals in both states. The change in orientation during a tensile test was the same in both cases. There was a very considerable difference in the yield-point and in hardening properties.

5. Gold-silver alloys

A series of gold-silver alloy crystals of different compositions has been prepared and tested by G. Sachs and J. Weerts.‡ These metals form a continuous series of solid solutions. There is a slight contraction in the lattice parameter at a composition of 50 atomic per cent.§ The mechanical properties of the crystals also show a variation consistent with a change of structure with composition. There is a marked increase in the shear stress on the slip-plane at the yield-point as silver is added to gold and gold to silver, with a maximum at 50 atomic per cent. (Fig. 47). There is also a tendency for slip to continue on the original slip-plane after

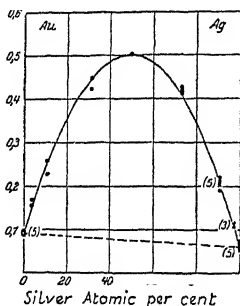


FIG. 47. Gold-silver alloy crystals.

the symmetrical position of two octahedral planes has been reached. Here again the maximum deviation is in the neighbourhood of 50 atomic per cent.

6. Aluminium alloys

Aluminium alloys|| containing 5 per cent. copper, 0.5 per cent. magnesium, and 5 per cent. copper plus 0.5 per cent. magnesium have been tested in the form of single crystals in order to investigate more fully the property of age hardening. The general type of deformation and fracture resembled pure aluminium and alloys of aluminium and zinc, and there was little difference between the annealed and hardened specimens, except in their mechanical properties. The shear stress at the yield-point was raised from 2 to 9.5 kg./mm.²; the stress at fracture rose from 20–25 kg./mm.² to 40–50 kg./mm.² On the other hand, the elongation was unaltered.

The experiments with alloy crystals indicate that the presence of

† G. Sachs, J. Weerts, *Z. Physik*, **67** (1931), 507.

‡ G. Sachs, J. Weerts, *ibid.*, **62** (1930), 473.

§ G. Sachs, J. Weerts, *ibid.*, **60** (1930), 481.

|| R. Karnop, G. Sachs, *ibid.*, **49** (1928), 480.

foreign atoms increases the resistance to shear and fracture, but not always with a corresponding loss of ductility.

7. Polycrystalline alloys

Most of the characteristics of alloy crystals are found again in normal polycrystalline metal. Although the usual effect of alloying is to reduce elongation and raise resistance to shear, an increased ultimate stress is sometimes accompanied by increased ductility. This sometimes occurs in copper-zinc[†] and copper-aluminium[‡] alloys. (Cf. also single crystals, p. 79.)

On the other hand the addition of copper[§] and zinc^{||} to aluminium raises both yield-point, ultimate stress, and reduces elongation (Figs. 48, 49).

Norbury has investigated the Brinell hardness of a number of α solid solutions, chiefly of copper.^{††} He has also measured the density of his alloys and compared the mean atomic volume with the atomic per cent. of added element.^{‡‡} When he compared the hardening effect of 1.0 atomic per cent. solute element with difference in size between solute and solvent atoms, the relationship was nearly linear (Fig. 51). He applied the same methods to determinations of Goebel^{§§} for tin and lead, and to values for copper- and zinc-aluminium alloys obtained by Rosenhain and Archbutt with like results.^{||||} There is thus a definite connexion between hardening properties of different solute atoms and the expanding or contracting effect that they have on the solvent.

According to Vegard,^{†††} the lattice constant is determined by the relative amounts of each element present, and if the two metals form a true series of solid solutions there is a linear relationship. This law has been partly confirmed by a large number of determinations of lattice constants by means of X-rays, to which it is impossible to refer here.

X-rays have also been the means of establishing^{‡‡‡} the fact that,

[†] F. Körber, Rohland, *Mitt. K. W. Inst. Eisenf.* **5** (1924), 55.

[‡] H. C. H. Carpenter, C. A. Edwards, *8th Alloys Res. Rep. Inst. Mech. Eng.* 1907.

[§] Carpenter and Edwards, loc. cit.

^{||} W. Rosenhain, S. L. Archbutt, *10th Alloys Res. Rep. Inst. Mech. Eng.* 1912.

^{††} A. L. Norbury, *J. Inst. Metals*, **29** (1923), 423.

^{‡‡} A. L. Norbury, *Trans. Faraday Soc.* **19** (1924), 586.

^{§§} J. Goebel, *Z. Metallkunde*, **14** (1922), 357.

^{||||} Rosenhain, Archbutt, loc. cit.

^{†††} L. Vegard, *Z. Physik*, **5** (1921), 17. L. Vegard, H. Dale, *Z. Krist.* **67** (1928), 440.

^{‡‡‡} Cf. work of Westgren, etc.

with few exceptions, the atoms of the added metal replace an equal number of those in the original lattice, producing an expansion or

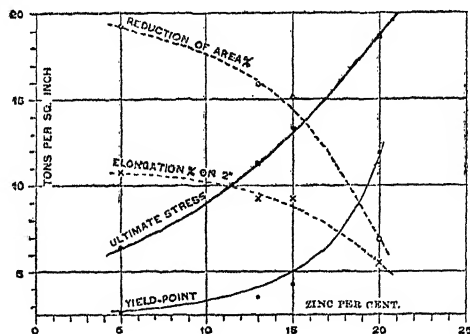


FIG. 48. Tensile tests on hot-rolled bars, $\frac{7}{8}$ -inch diam., after annealing at 400°C . (752°F .).

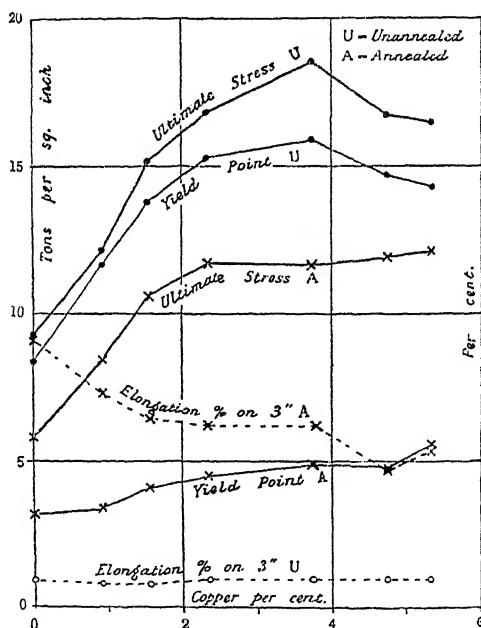


FIG. 49. Tensile tests on plates 0.05 inch thick.

contraction of that lattice. When the limit of replacement is reached there is generally a phase change, i.e. a new lattice appears which can accommodate the alloying atoms in a stable form.

In alloy systems which consist of solid solutions only, there is generally a maximum in the mechanical (and other) properties corre-

sponding to the approximate composition of a possible compound. Such a maximum occurs in the gold-silver alloys.† This may indicate a tendency towards a regular arrangement of the two kinds of atoms in the lattice,‡ or a closer bonding of the atoms, when in these proportions, in a manner similar to the bonding in compounds.

Speaking generally, it has not yet been established beyond doubt that the solute atoms take up any particular position in the lattice or, alternatively, are distributed at random. Probably both methods of substitution occur. Whichever is the case, there must be a distortion of the lattice in the neighbourhood of the stranger atom, and this has been compared with the distortion produced by mechanical means, owing to the similarity in the change of physical properties accompanying both forms of distortion.§

The hardening effect of alloy atoms has been explained on the assumption that the formation of slip-bands is interfered with because of the distorted lattice, and also, that when slip has begun, it cannot proceed far without irregularities in the slip-plane interlocking and preventing further movement.||

† W. Geibel, *Z. anorg. Chem.* **69** (1910), 38; **70** (1911), 240.

‡ This has been shown to occur in a number of alloy systems at compositions approximating those of possible compounds. For a discussion of these results see W. L. Bragg, E. J. Williams, *Proc. Roy. Soc. A*, **145** (1934), 699.

§ The change of physical properties produced by alloying is discussed by W. Rosenhain, *J. Inst. Metals*, **30** (1923), 3.

|| W. Rosenhain, *Proc. Roy. Soc. A*, **99** (1920), 192. Z. Jeffries, R. S. Archer, *Chem. Met. Eng.* **24** (1921), 1057.

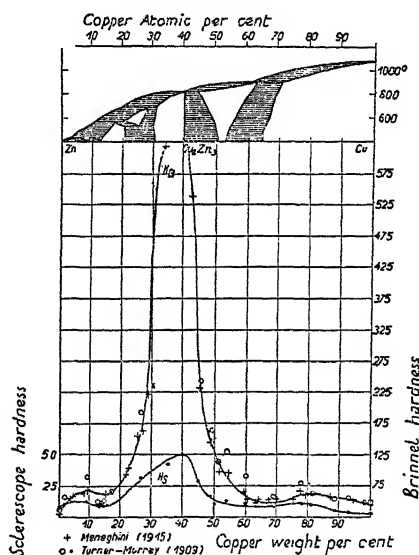


FIG. 50. Hardness of copper-zinc alloys.

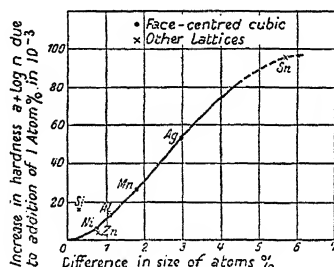


FIG. 51. Hardness of solid solutions of copper.

The distortion of alloy crystals described above does not support this theory entirely, as we find that in α brass for example, elongation before fracture may be greater than in copper. Reference is also made to the greater attraction between unlike atoms than like atoms.† This may be termed a chemical explanation, and may well account for the hardness of intermetallic compounds, but there is insufficient evidence of the nature of solid solutions to justify the analogy.

8. Intermetallic compounds

At ordinary temperatures compounds are usually brittle, and will often fracture in preference to deforming by gliding. The crystal structure of compounds is frequently complicated so that glide-planes do not form readily. Even those of cubic symmetry may have such a complicated structure that there is interlocking between every possible layer of atoms which might be expected to act as a glide-plane. Such crystals do not even fracture along a definite crystal plane. A typical example is the gamma phase of the copper-zinc, gold-zinc alloys, etc., and similar structures which are found in other systems. Mohs hardness numbers for a number of compounds are summarized by Kurnakow and Zemczuzny.‡

Certain compounds which are brittle at normal temperatures, have been found to be quite ductile at higher temperatures.§

9. Distortion of a series of alloys

The mechanical properties of a large number of alloy systems have been determined in addition to those mentioned above. The plasticity alters as the phase changes. The presence of a compound is indicated by a sudden increase in hardness and drop in ductility (Fig. 50).

10. Alloys consisting of more than one phase

The presence of even small amounts of a second constituent frequently hardens a metal considerably. The effect may be regarded as purely mechanical. Small particles of a harder, non-ductile material hinder both gliding and the formation of slip-planes. When the second constituent is soft, it has little effect.|| If the difference in hardness is

† G. Tammann, *Über die Beziehungen zwischen den inneren Kräften und Eigenschaften der Lösungen*, T. Voss, Leipzig, 1907.

‡ Kurnakow, Zemczuzny, *Z. anorg. Chem.* **60** (1908), 1.

§ Tammann and Dahl, *ibid.* **126** (1923), 104.

|| Cf. zinc-tin crystals, p. 74.

not great, both constituents deform together and there is a general stiffening of the material. A large number of cast metals, which have not been annealed long enough to remove coring, are common examples. Others are a 60-40 copper-zinc alloy consisting of the α and β phases, and slag in wrought iron. Both constituents are deformed approximately the same amount.

If the second constituent is much harder, it will remain almost unaltered in shape by the deformation. It will be pushed by the deforming, more ductile, metal into streaks and layers according to the type of distortion. Iron in aluminium, which exists in the form FeAl_3 , is always found in streaks of small particles in aluminium bars.

One of the most important examples of the effects of small quantities of a second constituent is that of iron carbide on the properties of ferrite. The mechanical properties of the steel are almost entirely dependent on the form of the pearlite. On very prolonged heating between 600 and 700° C. the cementite of the pearlite becomes globular. We no longer have alternate layers of the two constituents, but spherules of cementite are evenly distributed throughout the ferrite. Such a steel is more ductile, but tends to have a lower ultimate strength.

The following figures, due to Hanemann,[†] illustrate the effect in a eutectoid steel (0.9 per cent. C).

	kg./mm. ²	Extension %
Heated at 800° C. Cooled in air	89	12
Heated 6 hrs. at 670° C. Cooled in air.	68	20

A large number of other examples might be mentioned, such as the effect of cuprous oxide in copper, and the precipitation of compounds of copper and aluminium and magnesium silicide during the age hardening of duralumin. These have a beneficial effect on the mechanical properties. But it is also evident, that segregation of a hard, brittle substance in any position and particularly at the crystal boundaries will have a very deleterious effect. The addition of a small quantity of bismuth to copper or gold is a familiar example. Overheated metals, which become oxidized at the crystal boundaries, are also brittle. The usual strengthening effect of the crystal boundary is nullified by the presence of a brittle intercrystalline cement.

It has been impossible to do more than indicate very briefly some of the problems met with in the deformation of alloys. From what

[†] H. Hanemann, Fr. Morawe, *Stahl und Eisen*, 33 (1913), 1350.

has been said it is obvious that the structure of alloys can be very much altered by work, and this aspect of the subject is most important industrially. The effect of deformation is to mix the metal so that it becomes more homogeneous. It also breaks up any hard constituents which have separated in large crystals from the molten alloy. It is well known that, in certain cases, refining of the structure can never be obtained by heat treatment only. Plastic deformation followed by heat and recrystallization is the surest way of obtaining a homogeneous alloy.

VII

THE DEFORMATION OF METALS UNDER REPEATED STRESSES

IN the types of deformation already considered, the metal has been extended, compressed or twisted, until it fractures. It is now necessary to inquire into the nature of the deformation when the test is stopped before fracture takes place and the load is removed and re-applied, perhaps many times, before the test-piece finally fails. Such tests are commonly called 'Fatigue Tests'. They may take several forms.

1. Repeated loading and unloading in tension or compression.
2. Alternating tension and compression.
3. Alternating bending.
4. Alternating torsion.
5. Repeated impact tests.

A full account of the machines employed for such tests and the methods of testing is to be found in *Fatigue of Metals* by Gough.[†]

The early experiments of Fairbairn,[‡] and the more extensive ones of Wöhler[§], showed that metals would fail under repeated applications of stress of a less magnitude than would be required to cause failure under a single application of the same stress. The number of applications required to cause failure is greater, the smaller the stress. If, then, a number of test-pieces are subjected to applied stresses of increasing magnitude and the number of reversals of stress required to produce fracture determined, the values can be plotted and will give a curve similar to Fig. 52.

It will be seen that the curve tends to become parallel to the axis when the number of reversals is large. This indicates that, for the majority of metals, there is a limiting stress, below which fracture will not occur for an infinite or an indefinitely large number of reversals. This range of stress may be referred to as the 'safe' range.

It is unnecessary here to describe all the experimental evidence which has definitely led to the above conclusion, but similar curves have been obtained in all the types of test enumerated above.

[†] H. J. Gough, *Fatigue of Metals*, Scott, Greenwood & Son, 1924.

[‡] Sir W. Fairbairn, *Phil. Trans. Roy. Soc.*, 1864.

[§] A. Wöhler, *Z. Bauwesen*, vols. viii, x, xiii (1860).

We will now consider the manner in which the metal deforms under these conditions, and the accompanying change in the mechanical properties. Tests of this kind, carried out inside the elastic range, produce no permanent effect. Beyond the elastic limit there is a permanent deformation, and when the load is removed the metal does not return to its original form. This can be shown if a tensile test is carried out in stages and the load removed at each stage. Curves illustrating the phenomenon for zinc, tin, and bismuth†

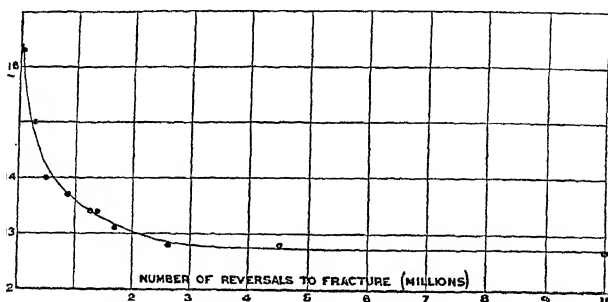


FIG. 52. Endurance tests (mild steel). Reversed bending stresses. (Stanton and Pannell.)

crystals are shown in Fig. 53, aluminium crystals in Fig. 56,‡ and for polycrystalline metal in Fig. 54.§ There are examples in these curves of both elastic and plastic deformation and combinations of both. There is in every case on first loading, a part of the curve which is approximately straight. The curve on unloading should coincide with the loading curve for purely elastic distortion. If the load is removed when plastic deformation has begun, the metal will contract a small amount elastically, but the unloading curve no longer coincides with the loading curve, the difference being a measure of the permanent deformation. On reloading, a new curve is traced, which usually makes a loop, the hysteresis loop, with the unloading curve. Such a loop will be found at every stage in the loading and unloading of the specimens whose curves are illustrated in Figs. 54 and 56.

The absence of the loops in Fig. 53 may be attributed to the small scale on which the results are plotted and also to the fact that the mechanical properties of metals of low melting-point, such as these,

† O. Haase, E. Schmid, *Z. Physik*, **33** (1925), 416.

‡ H. J. Gough, D. Hanson, S. J. Wright, *Phil. Trans. Roy. Soc. A*, **226** (1925), 1.

§ W. E. Dalby, *Strength and Structure of Steel and other Metals*. Arnold, 1923.

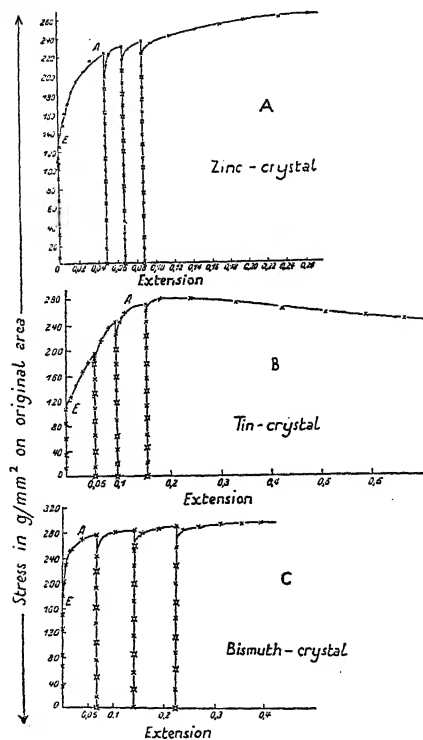


FIG. 53.

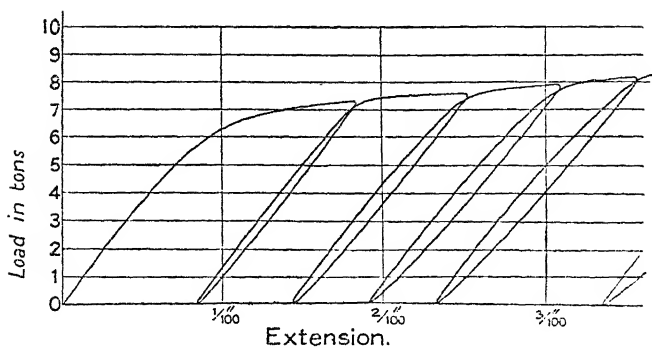


FIG. 54. Carbon steel.

alter even during the intervals of testing. On reloading after every extension, the elastic limit is raised nearly to the point of the previous load.

It frequently happens that a metal will continue to deform plastically under a given load. This phenomenon is known as 'creep' and becomes very important at high temperatures.†

Similarly, the elastic contraction which occurs on removal of the load does not always take place immediately. When it continues over a considerable period, it is known as 'after-working'. The application of heat hastens the movement.‡

All these phenomena are very much influenced by time and temperature, and by the rate of loading and unloading, etc.; and particular metals exhibit some of them more than others. But all of them occur to a greater or less extent and are intimately connected with the deformation.

Recovery after overstrain

If a metal is strained beyond the yield-point and is immediately re-tested, it will be found that the limit of proportionality is greatly reduced, sometimes to zero value. Its elasticity is slowly recovered on resting, and if it is again tested after a sufficient period is allowed to elapse, the limit of proportionality may even be higher than the value of the load producing the original overstrain. This phenomenon is very marked in iron and steel. Recovery is accelerated by heat-treatment, even at the temperature of boiling water.§ It is also reported|| that if mild steel and wrought iron are kept at a low temperature there is a partial recovery, after which all further recovery is arrested.

Bauschinger effect

Bauschinger†† was the first to show how easily the elastic limit of a metal could be altered by cyclical variations of stress. If a metal is subjected to tension beyond the yield-point and the elastic limit is raised thereby, and it is then compressed, the elastic limit in compression will be found to have a lower value than would be obtained on a first loading. In other words, if the elastic limit is raised by tension it is lowered for compression, and not necessarily to the same degree. Bauschinger called the elastic limits, which could be measured and found constant in a metal extended or compressed for the first

† Chapter IX.

‡ Chapter XV.

§ Muir, *Phil. Trans. Roy. Soc. A*, **193** (1900); **198** (1900).

|| E. G. Coker, *Phys. Rev.* **15** (1902), 107.

†† J. Bauschinger, *Ziviling.* **27** (1881), 289; *Mitt. Mechn. Techn. Labor. München* (1886), 13.

time, the 'primitive' elastic limits, and the new limits of proportionality produced by repeated cycles of tension and compression, the 'natural' elastic limits. He considered that a metal could withstand an infinite number of reversals of stress within the stress limits corresponding to the 'natural' elastic limits and we now refer to this as the 'fatigue' range. His theory was confirmed by the experiments of Bairstow.[†]

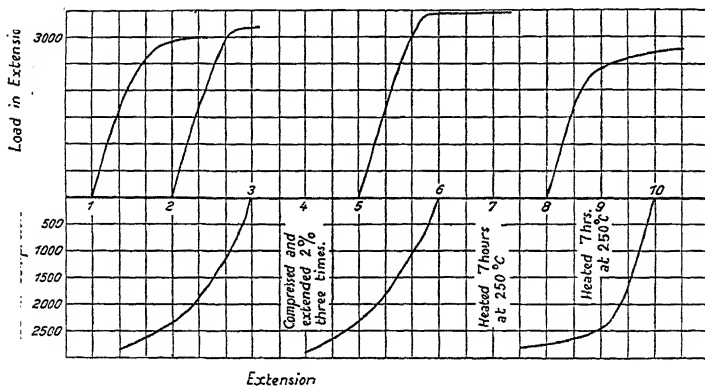


FIG. 55. Brass.

Sachs and Shoji[‡] have investigated the effects of small amounts of tension and compression on the form of the stress-strain curves of brass crystals when these are subsequently tested in compression and tension respectively. If the metal is slightly compressed and then tested in tension, or if it is first extended and then compressed, the curves are very similar. In both cases the elastic range is lowered and plastic deformation sets in with less load. They also found that heat treatment would restore the elastic properties without interfering with the strain hardness, and have suggested that in addition to increasing the resistance to shear, deformation has the effect of altering the properties in a directional manner.

Masing[§] has made a very extensive study of these phenomena and has also studied the effects of heat-treatment in restoring the metal to its original condition. Fig. 55 illustrates some of his results and shows very clearly the effect of tension followed by compression on the stress-strain curve.

[†] L. Bairstow, *Phil. Trans. Roy. Soc. A*, **210** (1910).

[‡] G. Sachs, H. Shoji, *Z. Physik*, **45** (1927), 776.

[§] G. Masing, *Wiss. Ver. Siemens-Konz.* **3** (1924), 231. G. Masing, also with W. Mauksch, *ibid.* **4** (1925), 244; **5** (1926), 142; **5** (1926), 135.

Repeated loading and unloading in tension and compression

Very careful experiments have been carried out by Gough, Hanson and Wright,[†] on aluminium crystals. They detected a permanent

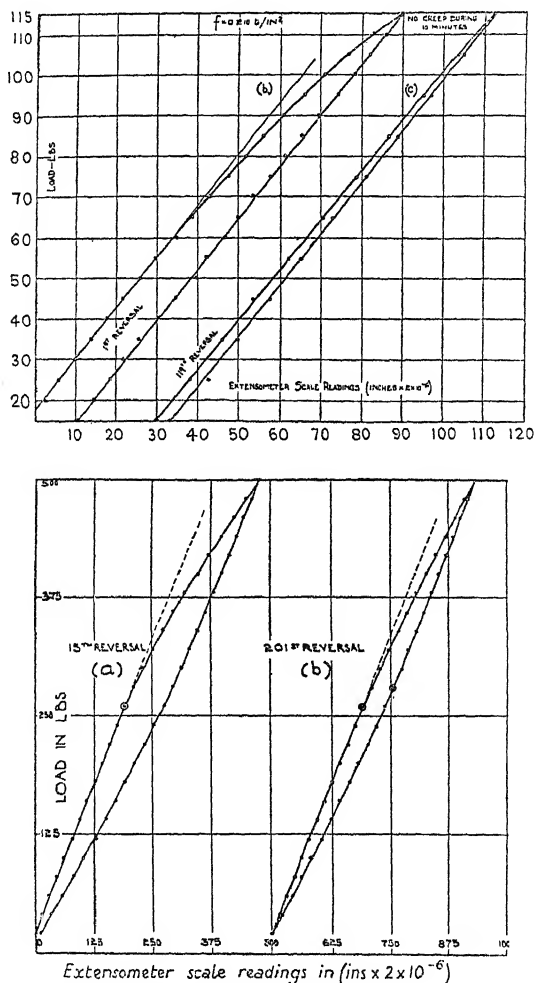


FIG. 56. Repeated loading of aluminium crystals.

extension after a load corresponding to only 0.073 ton/inch². They continued the experiment by loading and unloading the specimen until there was no further permanent set. When this was attained they increased the load and repeated the process. Whenever the load was

[†] H. J. Gough, D. Hanson, S. J. Wright, *Phil. Trans. Roy. Soc. A*, 226 (1925), 1.

increased the stress-strain relationship was almost linear up to the maximum load of previous cycles. The unloading curve, however, was not linear. Proceeding with the test a range of loading of 0.036–0.92 ton/inch² was used. It required some 200 reversals before a closed loop was obtained, i.e. before there was no further permanent set on loading. Curves obtained during this test are plotted in Fig. 56. They show the gradual closing up of the loop and the hysteresis loop in the cyclic state.

Experiments were then carried out to investigate the cyclic state under ranges of load whose maximum and minimum loads fall within the limits of the large cycle. Similar results were obtained as for polycrystalline metal, i.e. the metal exhibited elastic hysteresis without further permanent set.

Similar experiments with zinc, tin, and bismuth crystals are referred to above.

Alternating tension and compression direct stresses

Bairstow's† experiments were carried out in the following way. The specimen was subjected to direct tension and compression, the values of the range and mean stress of the cycle being adjusted as desired. The change in length of the specimen was measured by means of a mirror extensometer. The first range of stress was well under the value for the yield-point obtained in a static tensile test, and the first cycle loading showed the material to be elastic within the limits of accuracy of the experiment. On further repetitions of stress the straight line developed into a loop, the loading and unloading curves ceasing to coincide (Fig. 57).

The loading was in the direction *DEFG*, and *EF* and *GD* were found to be parallel to *ABC*. *FD* therefore represents an elongation which appears and disappears at each cycle and is referred to as the 'cyclical permanent set' and the total extension for each cycle is this amount, *FD*, plus the elastic extension. For every stress range the cyclical permanent set approaches a constant value, and when these values are plotted against the applied stresses an approximate straight line is obtained which intersects the stress axis at a positive value. Bairstow concluded that a range of stress was thus indicated under which the material would possess or acquire a state of perfect elasticity under indefinite repetitions of the stress range, i.e. that the

† Bairstow, loc. cit.

fatigue range is an elastic range. He thus confirmed Bauschinger's hypothesis. In all such tests Bairstow found no permanent extension of the specimen.

In the preceding experiments the stresses in tension and compression were equal and at one point in the cycle there was zero stress. Similar experiments can be carried out, in which there is always a tensile (or compressive) mean stress. Repetitions of all ranges of stress, whose superior limit exceeds the superior limit of the safe

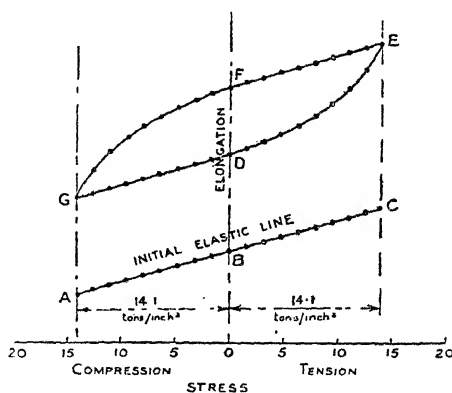


FIG. 57.

range for reversed (equal) stress, produces a permanent extension. For example in Bairstow's experiments, an axle steel test-piece (ultimate strength 38.2 tons/inch^2) was tested so that the stress at the superior limit was $+34.4 \text{ tons/inch}^2$, and at the lower limit $+18.9 \text{ tons/inch}^2$. There was a large permanent extension at first which finally ceased, and the initial large amount of cyclical permanent set also diminished, so that after 6,000 reversals, the metal was almost elastic. The range was then increased by reducing the lower limit to $+15.3 \text{ tons/inch}^2$. No change was observed, but when the range was increased still farther, i.e. $+7.3$ to $+34.4$ tons, the metal began to yield once more. In other words, with any superior limit of applied stress, there is a corresponding inferior limit of stress below which the material must not be unloaded, if failure is to be avoided. Another very interesting fact was established through Bairstow's experiments. He compared the total elongation obtained in statical loading with that produced under repeated stress. He showed that when the cyclic state is attained, the total extension at the superior

stress under alternating stresses is approximately equal to the extension obtained under the same stress in a direct tension test.

Alternating bending and torsion tests

The classical experiments of Wöhler are examples of alternating bending tests, and alternating torsion tests have also been carried out in metal in both polycrystalline and single-crystal form. In both forms of test there is no increase in length of the specimen. References will be found in the bibliography and in the section dealing with single crystals. The main result of all these tests is the same and may now be summarized as follows. There are three classes in which the results may be grouped:

1. The safe range of reversed stress is within the primitive elastic limits.
2. The safe range of reversed stress exceeds the range of the primitive yield-points.
3. There is no primitive elastic limit, but the metal exhibits a clearly defined safe range of stress.

From a general consideration of the tests the following facts are established:

1. During the first stage of the test; (*a*) relatively large hysteresis loops are observed whose widths tend to diminish with subsequent cycles; or (*b*) a loop is formed and attains a certain width.
2. During the second stage (which may extend over millions of cycles); the loop may maintain a constant width, or may slowly increase or decrease.
3. During the third stage; the rate of increase of permanent set and of cyclical permanent set usually increases and fracture results.

The first two stages are common to both safe and unsafe ranges of stresses; the third is almost certainly associated with the formation of cracks and break-down of the specimen with unsafe ranges of stress.

Changes in the microstructure accompanying fatigue tests

Ewing and Humfrey† first described the changes in the microstructure of iron under fatigue stresses. They used rectangular bars, of which one surface was polished, and a small Wöhler machine. They observed

† Ewing, Humfrey, *Phil. Trans. Roy. Soc. A* (1903), 241.

the appearance of slip-bands, their increase in number and tendency to broaden. After a number of reversals, the broadening increased and cracks formed in the broad groups of slip-bands. They put forward the theory that, as soon as slip begins, the metal is bound to fail by a process of attrition on the slip-planes. The metal is

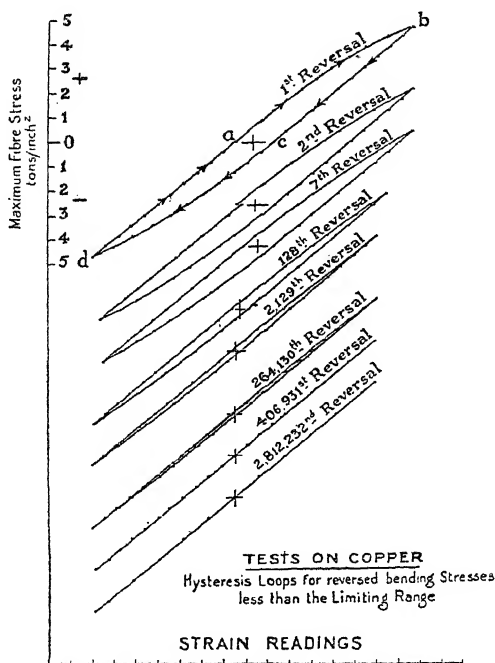


FIG. 58.

pushed backwards and forwards, and is gradually weakened on the slip-planes, until fracture takes place.

Baird's† work, however, showed that a metal could acquire an elastic state, even after it had been deformed plastically. Hence the metal was actually strengthened by being subjected to reversed stresses. Gough and Hanson‡ then undertook a more careful study of the microstructure in both safe and unsafe ranges of stress in armco iron, steels, copper, etc. At stresses below the value of the limit of proportionality for the static tensile test, there were very slight markings. These were quite noticeable at stresses correspond-

† Baird, loc. cit.

‡ H. J. Gough, D. Hanson, *Proc. Roy. Soc. A*, 104 (1923), 538.

ing to the range between the limit of proportionality and the yield-point. At stresses beyond the yield-point, but within the fatigue range, a large number of fine slip-bands formed and these did not differ in character, but only in number, from bands formed by stresses in the unsafe range. The dark broad bands described by Ewing and Humfrey were found to consist of a large number of very fine lines. They concluded that slip in the crystals actually produces a hardening effect (as was proved to be the case in static tests) and that slip, having once occurred on certain planes, tends to take place on others next time, thus increasing the *number* and consequent *fineness* of the observed slip-bands. They were unable to obtain any evidence with regard to the actual mechanism of cracking, but when it had once begun, it spread, due to stress concentration at the end of the crack.

Another interesting point was cleared up by these experiments. A substance such as annealed copper,† which possesses no primitive elastic limit, may acquire a cyclic state and even becomes elastic by repeated stresses (Fig. 58). This result may be compared with those obtained on repeated loading of aluminium crystals already described. Quite clearly the formation of slip-bands did not lead to a mechanical weakening of the crystal, but the metal became more able to bear further repetitions of stress.

Fatigue tests on single crystals

Reference has already been made to the effect of loading and unloading a crystal of aluminium. Gough, Hanson, and Wright‡ also carried out tests employing alternating direct stresses and alternating torsion. In the former, the crystals generally extended and the deformation was the same as in a static tensile test. Slip-bands were observed after the first applications of stress, however small. Within the safe range (which is very low) the number of slip-bands produced decreases very rapidly and finally ceases. The metal will then exhibit a state of elastic hysteresis which will persist indefinitely. The final pulling down and fracture of the specimen was exactly similar to results obtained in a normal tensile test.

Crystals subjected to reversed torsional stresses

In this form of testing, a crystal cannot possibly deform by shear on any one set of planes, without involving discontinuities in the lattice,

† H. J. Gough, *Fatigue of Metals*, p. 129.

‡ H. J. Gough, D. Hanson, S. J. Wright, *Phil. Trans. Roy. Soc. A*, 226, 1.

which is known from X-ray analysis not to occur. One can, however, calculate the shear stress on all possible slip-planes, and for all possible directions of slip, round the test piece and compare them with the slip-bands which are formed at corresponding points.

This has been done for aluminium,[†] iron,[‡] zinc,[§] antimony,^{||} silver,^{††} etc.

1. Aluminium and silver

In one aluminium crystal, X-ray analysis showed that two {111} planes were nearly parallel to the axis. On testing, slip-bands were developed, also parallel to the axis, and were concentrated at four points, nearly at 90° to each other. Fracture finally took place by the formation of a number of longitudinal cracks, parallel to the slip-bands. There was no increase in length during the test, and very little distortion. At the end of the test only 4° twist was measured between the two ends. When a section was cut and the surface polished, a system of fine markings was revealed, forming a 'herring-bone' pattern, which divided the section into four quadrants. These markings appeared to be due to differences in hardness shown up by the polishing. They agreed in directions with possible octahedral planes, and putting all the evidence together it was concluded that deformation had taken place by slip on the two octahedral planes which, as already mentioned, were parallel with the axis, and whose traces were represented by slip-bands on the surface and by the fine markings on the cross-section. Further, that one octahedral plane accounted for the markings in opposite quadrants, and the second octahedral plane in the other pair of opposite quadrants. An analysis of the shear stress on each octahedral plane in each of the three directions corresponding to the principal lines of atoms (i.e. [110] directions) was made. It can be shown^{††} that the 'resolved' shear stress in any one plane, in a certain direction, varies at different positions round the circumference of a round specimen, according to the following relationship:

$$S_r/S = A \cos(\lambda - \alpha),$$

[†] H. J. Gough, D. Hanson, S. J. Wright, *Phil. Trans. Roy. Soc. A*, **226**, 30; *J. Inst. Metals*, **36** (1926), 173.

[‡] H. J. Gough, *Proc. Roy. Soc. A*, **118** (1928), 498.

[§] H. J. Gough, H. L. Cox, *Proc. Roy. Soc. A*, **123** (1929), 143; **127** (1930), 453. W. Fahrenhorst, E. Schmid, *Z. Metallkunde*, **23** (1931), 323.

^{||} H. J. Gough, H. L. Cox, *Proc. Roy. Soc. A*, **127** (1930), 431.

^{††} H. J. Gough, H. L. Cox, *J. Inst. Metals*, **45** (1931), 71.

^{††} H. J. Gough, D. Hanson, S. J. Wright, *J. Inst. Metals*, **36** (1926), 173.

where

S_r = resolved shear stress,

S = maximum shear stress on specimen,

= $2T/\pi r^3$, where T is the applied torque and r is the radius of the cross-section of the specimen,

$A = \{-\frac{1}{2}(\cos 2\theta_p + \cos 2\theta_d + 2 \cos 2\theta_p \cos 2\theta_d)\}^{\frac{1}{2}}$,

$\alpha = \tan^{-1}[(\tan^2\theta_p - 1)\cot(\psi_d - \psi_p)] + \psi_p$,

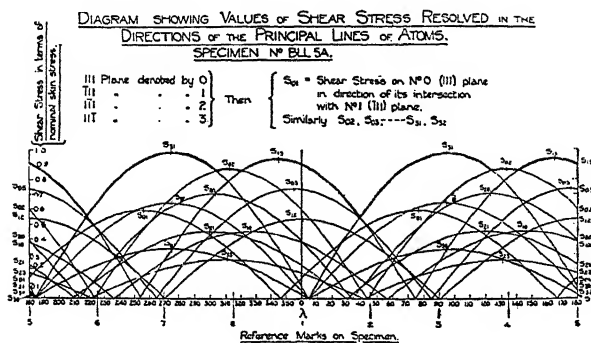


FIG. 59. Results of complete shear stress analysis of a single crystal of aluminium subjected to a torsional couple.

and θ_p , ψ_p and θ_d , ψ_d are the spherical coordinates of the plane and contained direction, respectively, and λ = angle from reference line round circumference of specimen.

Fig. 59 shows the results of such a series of calculations in the case of an aluminium crystal plotted in relation to the circumference, i.e. against λ . The resolved shear stress on some planes is much greater than on others and reaches maxima at definite values of λ . It is at these regions on the surface of the test-piece that we should expect to find slip-bands, and when one curve gives place to another, a change of slip-plane is involved, and consequently a new series of slip-bands appears on the surface. An abrupt change over of slip-bands is illustrated in Fig. 4, Plate V, but frequently the change is more gradual. The general formation of slip-bands, however, agrees with the stress distribution as calculated above, and the results of these experiments are in agreement with those obtained in other ways.

It was also observed that the main cracks started from positions close to the points of maximum shear stress. A very interesting observation was the absence of signs of slip near the start of the

fracture. The authors conclude that fracture is due to the propagation of discontinuities which are formed during the earlier stages of the test.

It may be mentioned that in carrying out these tests very small loads were used and a very large number of reversals. In this way the maximum hardening effect was obtained, as the process was more gradual and a greater number of intermediate stages could be investigated. The number of reversals required to fracture one specimen was 9,274,900, at a stress of ± 1.0 ton/inch².

The markings found on polishing a cross-section of the test-piece were thought to be indicative of preferential hardening on the slip-planes. Brinell impressions were made at different points and compared with the calculated shear stress.

Number of Impression	Shear Stress	Hardness Ratio
1	0.886	1.37
2	0.645	1.10
3	0.000	1.00
4	0.806	1.33
5	0.789	1.23
6	0.561	1.17
7	0.844	1.28
8	0.442	1.14
9	0.306	1.17

There is a general agreement indicating a close relationship between intensity of shear stress at a point and hardness. It should be mentioned that Brinell impressions on single crystals are always irregular in shape, so that the hardness ratio is only a comparative figure.

A silver crystal[†] was treated in the same way particularly with a view to studying twin formation. In a tensile test,[‡] silver crystals behave like other crystals of a face-centred cubic lattice, and slip-bands were also formed in this experiment corresponding to traces of {111} planes in good agreement with the maximum resolved shear stress law. No twinning was observed.

2. α iron

Methods similar to those employed in aluminium crystals were again used for α iron.[§] The distortion of iron (and also β brass, which has

[†] H. J. Gough, H. T. Cox, *J. Inst. Metals*, **45** (1931), 71.

[‡] C. F. Elam, *Proc. Roy. Soc. A*, **112** (1926), 289.

[§] H. J. Gough, *Proc. Roy. Soc. A*, **118** (1928), 498.

the same crystal lattice) in tension and compression has been described in Chapter II. It has been proved that the direction of slip is parallel to the normal of a $\{111\}$ plane, but the pole of the plane of slip may be that of any of the planes at 90° to this direction and was in fact found to vary between a $\{110\}$ or $\{112\}$ plane according to the stress distribution. In support of this, observations on the character of the slip-bands showed that, on a section parallel to the direction of slip, the slip-bands were straight, but in directions at 90° to this they were broken and wavy, the main direction agreeing with the trace of the slip-plane as determined from distortion measurements.

Gough confirmed the direction of slip parallel to a $[111]$ direction and also showed that it agreed with the most highly stressed principal line of atoms. He also showed that the average slope of the slip-bands at any point approximated to the slope of the trace of the plane of maximum resolved shear stress. Examination of the slip-bands themselves showed that they were of three types:

1. Straight.
2. Wavy; but on closer investigation they could be resolved into two sets making small angles with each other.
3. Wavy; but not resolved as in 2.

Now, the poles of crystal planes perpendicular to a $[111]$ direction which may be involved in the distortion will be of the type $\{110\}$ $\{112\}$ $\{123\}$ $\{134\}$, etc., and at certain points on the circumference where the plane of maximum resolved shear stress coincided with one of these planes the slip-bands on the surface agreed with the calculated traces of the particular plane.

If, however, the plane of maximum resolved shear stress were situated between two planes of the type $\{110\}$ and $\{123\}$ or $\{112\}$ and $\{123\}$, etc. (it is unlikely that planes of higher indices are involved), the distortion might be expected to be made up of slip on both planes, the relative amounts being adjusted so as to produce the required distortion on the plane of maximum resolved shear stress. Slip-bands which appeared on the surface at these points were always wavy and in some cases their duplex nature was sufficiently pronounced to enable the directions of the two components to be measured. The general inclination of the bands agreed with the plane of maximum resolved shear stress.

Markings on a cross-section, such as were found in aluminium

crystals, were not observed, but etching revealed zones corresponding to the directions of slip in each section of the test-piece.

3. Zinc

Zinc crystals† were treated in the same way as aluminium and iron crystals. The slip-plane and direction of slip were found to be the basal plane of the hexagonal close-packed lattice (0001) and first order prismatic (1010) respectively, confirming the work of Mark, Polanyi, and Schmid. Contrary to their work, but confirming the observations of Mathewson and Phillips and Schmid and Wassermann,‡ neither slip nor fracture occurred on any prismatic plane. Twins were formed on planes of the {1012} form. These twins deformed by slip on their basal planes also. Fracture followed three directions:

1. Parallel to the traces of the original basal plane.
2. Parallel to the traces of the basal planes in the twins.
3. Along the edges of the twins.

A description of the possible causes of twinning as revealed in these experiments will be found in the section on deformation by twinning. Deformation by slip and formation of twins were determined by resolved shear stress considerations.

4. Antimony

The lattice of antimony is very nearly cubic. The crystal is rhombohedral, of trigonal symmetry. The deformation in tension is not known. Gough and Cox§ state that no slip-bands were observed in their tests. Fine markings on the surface which were parallel to traces of a {111} plane of the original crystal became identified with cracks later on. Twinning was common and was observed on all three planes of the type {110}. These again formed secondary twins, also of this type. Final fracture occurred by cleavage parallel to one of the twinning planes.

5. Bismuth

Similar tests on bismuth crystals|| failed to produce any slip-bands whatever, although twins formed readily. Other markings were

† H. J. Gough, H. L. Cox, *Proc. Roy. Soc. A*, **123** (1929), 143; **127** (1930), 453.

‡ See Chapter IV on Twinning.

§ H. J. Gough, H. L. Cox, *Proc. Roy. Soc. A*, **127** (1930), 431.

|| H. J. Gough, H. L. Cox, *J. Inst. Metals*, **48** (1932), 227.

found but these were not identified as either slip-bands or twins. The investigators found that twins were produced on planes subjected to a high shear stress, whereas the conclusions at which they arrived in the case of zinc rather suggested a greater influence of the stresses normal to the twin plane.

VIII

FRACTURE

MANY metals fracture without any plastic deformation, i.e. their elastic limit, yield-point, and fracture all occur simultaneously. Examples of these have been referred to already in zinc and bismuth crystals of certain orientations and tellurium crystals. Most metals deform plastically to a greater or lesser extent before fracture occurs.

The factors governing fracture are not at all certain. It is well known, for example, that the rupture stress of metals and solids in general is very much less than would be expected from calculations of their intrinsic pressure.† The nearest approach to theoretical strength was that obtained by Griffith‡ in glass and silica rods. This discrepancy is one of the problems of cohesion which will be discussed in another connexion.

In metal crystals fracture occurs on one or more planes which may or may not be glide-planes. In iron the cleavage plane is $\{100\}$ and the slip-planes of type $\{110\}$ or $\{112\}$.§ This is also the case in a large number of minerals, e.g. rock-salt, in which the cleavage plane is of the type $\{100\}$ and the glide-plane of type $\{111\}$. It was shown by Schmid|| that in zinc crystals, whereas the yield-point depends on the value of the shear stress on the glide-plane, fracture was more closely related to the stress normal to the plane. In zinc and cadmium both glide-plane and plane of fracture are the same crystallographically, i.e. the basal plane. Fig. 60 shows the relation between breaking strength of zinc crystals at -180°C . and the angle which the plane of fracture makes with the axis. If now the stress normal to the plane is calculated, it will be found to have an approximately constant value, 180 gm./mm.^2 in the case of zinc, whatever the inclination of the plane to the axis. The shear stress, on the other hand, varies considerably from crystal to crystal depending on the angle which the slip-plane makes with the axis, and on the direction of slip. In bismuth†† at -80°C ., where there is no plastic deformation before fracture, the constancy of the normal stress on the fracture plane is

† Traube, *Z. anorg. Chem.* **34** (1903), 413.

‡ A. A. Griffith, *Phil. Trans. Roy. Soc. A*, **221** (1921), 163.

§ C. A. Edwards, L. B. Pfeil, *J. Iron and Steel Inst.* **109** (1924), 129.

|| E. Schmid, *Proc. 1st Intern. Congr. Appl. Mech.* Delft, 1924.

†† M. Georgieff, E. Schmid, *Z. Physik*, **36** (1926), 759.

even more noticeable. This brings us to another point. Slip on a plane produces increased resistance to shear on that plane, and

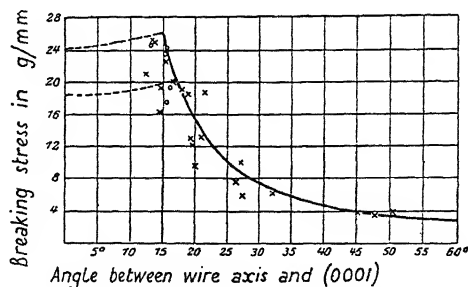


FIG. 60. Zinc. Breaking strength of zinc crystals at -180°C . in relation to the orientation.

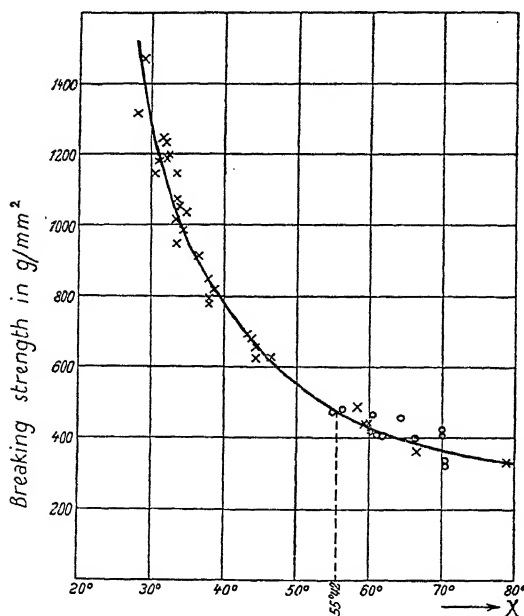


FIG. 61. Breaking strength of bismuth crystals in relation to orientation. χ = angle between glide plane and axis of wire.

Schmid† found that the stress normal to the slip-plane and plane of fracture was also increased by deformation. He not only showed that the breaking stress increased with the amount of deformation, but

† E. Schmid, *Z. Physik*, 32 (1925), 918.

also that it decreased if the specimen was allowed time for recovery at a higher temperature.

The effect of temperature on the breaking stress is similar to its effect on the yield-point, tensile stress, etc. (Chap. IX).

Tellurium[†] behaves similarly when the basal plane (which is the plane of fracture) makes a larger angle with the axis than 30° . If it is more acutely inclined, fracture occurs approximately at 90° to the axis and is uneven. The stress normal to the basal plane is much less than the constant value, but the breaking stress remains about the same.

Still less is known about the fracture of metals which deform plastically a considerable amount before fracturing. Amongst these are two types, (1) those which show a well-marked cleavage fracture, e.g. zinc and cadmium at normal temperatures, aluminium alloys, etc. (2) those which draw down to a wedge and have no plane of fracture, e.g. aluminium, copper, etc.

It has been pointed out that the constancy of the stress normal to the plane of fracture does not hold when much distortion takes place before fracture. The hardening produced by slip also affects the stress required to fracture. This is also clearly shown in aluminium-zinc alloy crystals,[‡] as the more they extend the higher their breaking stress. It is significant that fracture usually takes place when the slip-plane is approximately 45° to the axis, i.e. when the shear-stress on this plane is at a maximum. In all these crystals fracture coincided with the slip-plane $\{111\}$ or was at 90° to it along the corresponding $\{112\}$ plane. The evidence is such that one can predict that a test-piece will fracture if (1) it becomes non-uniform in cross-section, (2) the rate of increase of resistance to shear is less than the rate of application of the load.[§] Beyond this, we know practically nothing about the conditions governing fracture and any explanation must be sought finally in the realm of inter-atomic forces.

[†] E. Schmid, G. Wassermann, *Z. Physik*, **46** (1928), 653.

[‡] C. F. Elam, *Proc. Roy. Soc. A*, **115** (1927), 133. See Chapter VI.

[§] G. I. Taylor, C. F. Elam, *Proc. Roy. Soc. A*, **108** (1925), 28.

IX

EFFECT OF TEMPERATURE, DURATION OF TEST, AND RATE OF APPLICATION OF LOAD

THE temperature at which a test is made and the rate at which the metal is deformed are closely related. The rate of testing becomes most important at higher temperatures because of the tendency to recrystallize caused by straining. This means that the effects of deformation are removed as soon as they appear, provided the straining is sufficiently slow and the temperature sufficiently high. The temperature at which recrystallization takes place depends on the metal, particularly on its melting-point and on the amount of deformation. This problem is referred to more fully in the section on the effects of heat after deformation.†

The three factors determining the results of all mechanical tests on metals are: (1) temperature, (2) amount of previous deformation, (3) duration of test. It is impossible to consider any one of these without reference to the other two. It is proposed to discuss both (1) and (3) together in relation to (2), the effects of which have been dealt with already. It has been assumed, up to the present, that the conditions under which deformation was carried out were suitable to produce the structure and alter the properties in the manner described, i.e. that the temperature was too low to affect the results to any extent. The lower the temperature the less the effect of time, so that in these circumstances both (1) and (3) are less important. All the effects of deformation on the mechanical properties which have been described—with the exception of purely geometrical considerations—are essentially those associated with what is called ‘cold working’. It will be seen in the following pages that many of the changes are similar though different in degree, and that the higher the temperature and the slower the rate of deformation, the less the tendency for the effects of work to persist.

1. Tests on metal crystals at high temperatures

Some early work of Schönborn‡ on tungsten single crystals carried out at high temperatures showed that the rate of deformation was an

† See Chapter XIV.

‡ H. Schönborn, *Z. Physik*, 8 (1922), 377.

exponential function of the load.† Goucher's‡ work was more extensive and was confined to results from crystals similarly orientated to the stress. His results were in agreement with Schönborn's. He found that the time required for fracture to take place, which may be regarded as an inverse measure of the mean rate of deformation, is approximately an exponential function of the load and the temperature. There were well-defined breaks in the curves at certain temperatures, but between these temperatures the load was proportional to the cross-sectional area for a given mean rate of deformation and was a linear function of the temperature.

The original investigation by Mark, Polanyi, and Schmid§ of the extension of zinc crystals showed that certain crystals which could be extended easily at room temperature and up to 200° C. were extremely brittle at the temperature of liquid air, and that the end-angle of the basal plane varied with the temperature.

A number of investigations have since been made on the influence of temperature and rate of deformation on metal crystals.

Polanyi and Schmid|| compared the yield-point of tin crystals on extending for the first time and after they had been broken as rapidly as possible. The results were as follows:

Before breaking		After breaking	
Yield-point mg./mm. ²	Extension per cent.	Yield-point mg./mm. ²	Extension per cent.
340	350	1,100	..
307	400	1,610	..
252	430	785	5

Further tests were made at the temperature of liquid air. The rate of extension is very important in the case of tin, which will recrystallize at room temperature. It is also to be expected that a different result will be obtained if the metal is tested immediately after working or if it is allowed to rest for a day.

Bismuth,†† cadmium,‡‡§§ and zinc§§||| crystals have been tested

† See also W. Geiss, *Z. Physik*, **29** (1924), 78.

‡ Research Staff of the General Electric Co. (F. S. Goucher), *Phil. Mag.* **48** (1924), 229.

§ H. Mark, M. Polanyi, E. Schmid, *Z. Physik*, **12** (1923), 58.

|| M. Polanyi, E. Schmid, *Z. Physik*, **32** (1925), 684.

†† M. Georgieff, E. Schmid, *Z. Physik*, **36** (1926), 759.

‡‡ W. Boas, E. Schmid, *ibid.* **56** (1929), 516; **57** (1929), 575.

§§ W. Boas, E. Schmid, *ibid.* **61** (1930), 767.

||| W. Fahrenhorst, E. Schmid, *ibid.* **64** (1930), 845.

at different temperatures. All these crystals have one slip-plane only, whose inclination to the wire axis determines whether the crystal is ductile or brittle and also determines the shear stress at the yield-point, or, in the case of brittle crystals which fracture without extension, the stress required to fracture. These values are very much

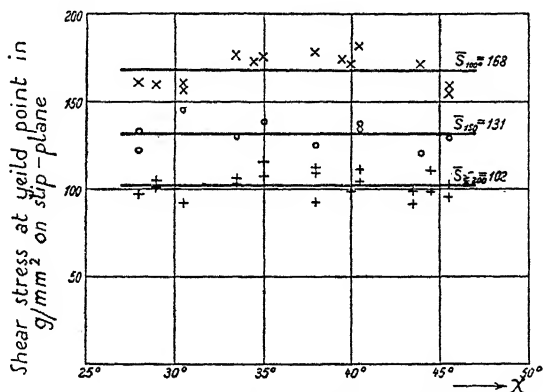


FIG. 62. Bismuth crystals tested at three temperatures. χ = angle between glide plane and wire axis.

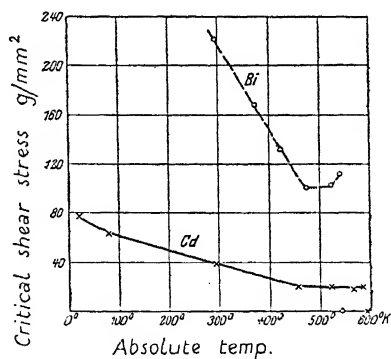


FIG. 63.

influenced by the temperature at which the metal is tested. Fig. 62 shows that bismuth crystals of different orientation yield at the same shear-stress value at the same temperature. The value decreases as the temperature increases. In Fig. 63 are plotted the results obtained on testing bismuth and cadmium crystals at different temperatures. Fig. 64 shows the shear stress plotted against amount of shear for a number of cadmium crystals. It also shows the effect of slow and

fast loading on the shape of the curves. The influence of rate of extension in relation to temperature is brought out in Fig. 65. The critical shear stress is higher for all temperatures at the highest rate

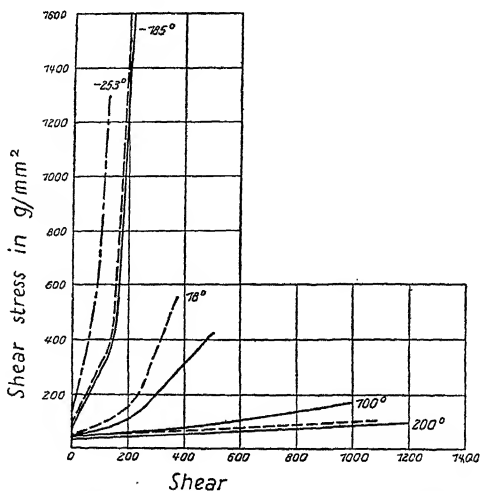


FIG. 64. Cadmium crystals tested at different temperatures.

— Rate of loading 10 gm. 10 sec.
 --- Duration of test 12 sec.

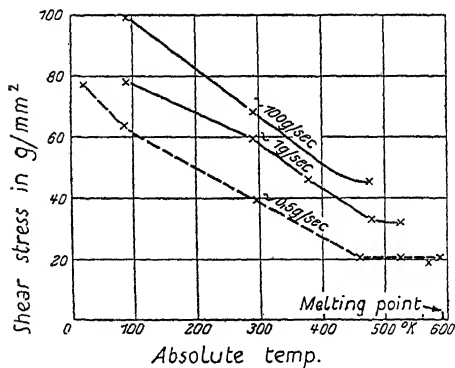


FIG. 65. Cadmium crystals tested at different temperatures and rates of extension.

of loading. It may be noted that beyond a certain temperature, 473° K., there is no further drop in shear stress with increasing temperature.

The tangents of the angles which the straight portions of the curves in Fig. 64 make with the ordinates is a measure of the hardening for

each temperature and may be called the 'coefficient of hardening'. At high temperatures it is very small, as the tendency to soften is almost equal to the hardening due to the deformation.

In a previous investigation,† it was shown that the amount of shear on the basal plane was limited by the formation of twins. Twin formation always took place when the angle between the glide plane and direction of pull was approximately 4° to 7° at normal temperatures, quite independently of the original orientation of the crystal. Boas and Schmid‡ find that there is a fixed limit to the amount of shear for each temperature, independent of the original orientation. The work done in extending a crystal is thus also independent of the orientation, and since the components of shear are those concerned in this type of deformation, the work done by shear is equal to that done in extending a crystal for all orientations. On these assumptions they find the energy to vary in the following way:

	-253° C.		-185° C.		18° C.		100° C.	200° C.	
Work done in shear Joule/cm. ³	slow	fast	slow	fast	slow	fast	slow	slow	fast
	6.36	6.64	9.14	9.97	9.18	7.35	9.42	7.67	8.06

These figures refer to the amount of work done, up to the point when slip on the basal plane ceases and twins are formed. The difference between fast and slow testing is more marked at high temperatures, at which softening is also more rapid. Allowing for experimental errors, there is very little difference between the work done in extending a cadmium crystal at -253°C. and at 200°C. In zinc, there is a maximum at 150°C. , although otherwise the results for the two metals are similar.

Certain crystals which are brittle, because of the position of the slip-plane in relation to the axis, fracture without extension. The temperature determines the stress normal to the slip-plane at which fracture occurs.§

Tensile tests on aluminium crystals at high and low temperatures have also been made.|| The distortion takes place in the same way as at normal temperatures. According to Boas and Schmid,†† other

† W. Boas, E. Schmid, *Z. Physik*, **54** (1929), 16.

‡ W. Boas, E. Schmid, *ibid.* **61** (1930), 767.

§ See Chapter VIII on Fracture.

|| W. Boas, E. Schmid, *Z. Physik*, **71** (1931), 703. K. Yamaguchi, S. Togino, *Sci. Papers Inst. Phys. Chem. Res. Tokyo*, **9** (1929), 277. D. Hanson, M. A. Wheeler, *J. Inst. Metals*, **45** (1931), 229.

†† W. Boas, E. Schmid, *Z. Physik*, **71** (1931), 703.

planes become involved above 400°C . Slip was observed on the cube planes in a $[110]$ direction, and the end orientation of the crystal was such that a $[111]$ direction was in the axis of the test-piece (instead of the usual $[112]$). Yamaguchi and Togino[†] do not report this, and they made tests up to 525°C . A series of typical stress-strain curves for crystals tested at different temperatures is shown in Fig. 66. There is a very large increase in the hardening at low temperatures.

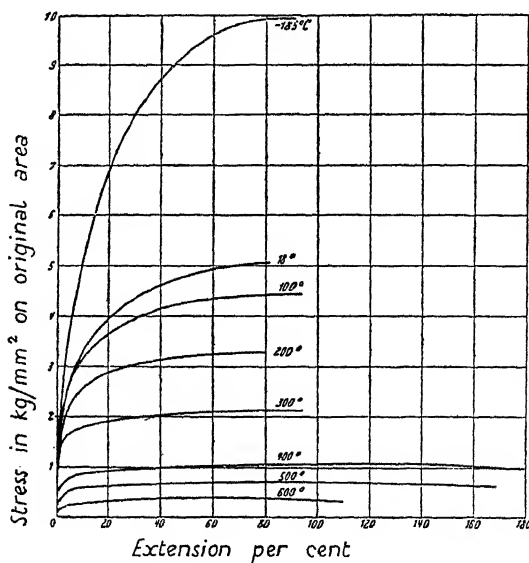


FIG. 66. Aluminium crystals tested at different temperatures.

Yamaguchi estimated the number of slip-bands for the same amount of shear and found that a larger number were produced in liquid air, from which he concluded that the amount of shear possible on a given plane is less at low temperatures than it is at high, so that in order to obtain the same amount of total shear, more slip-planes must take part in the deformation. This observation is in agreement with photographs of zinc and cadmium crystals pulled at high temperatures shown by Boas and Schmid.

One very interesting fact was pointed out by Yamaguchi and Togino. A crystal which has been extended at high temperatures and showed no recrystallization, would, if extended the same amount

[†] K. Yamaguchi, S. Togino, *Sci. Papers Inst. Phys. Chem. Res. Tokyo*, **9** (1929), 277.

at normal temperatures, recrystallize if subsequently heated to the higher temperature. This provides very interesting evidence of the recovery which is continually taking place during deformation above a certain temperature.

Tests on zinc and cadmium crystals have been carried out in liquid oxygen† (20° T. abs.) and also liquid helium‡ (1.2° T. abs.). At these low temperatures the yield-point alters only to a small

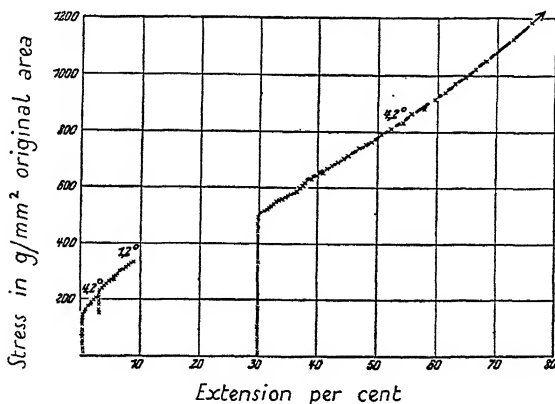


FIG. 67. Cadmium crystal tested at low temperatures.

extent compared with other properties. The following figures are given by Polanyi and Schmid:

Yield-point of Zn and Cd crystals at low temperatures

Temperature (absolute)	Shear stress at yield-point in gm./mm. ²	
	Zn	Cd
300°	109, 114	70 (mean of 5)
83°	140, 122	110, 70
20°	188, 166, 183 (mean 153)	75

For cadmium the value is nearly constant.

From 20° to 12° T. abs. there is also little change. The hardening coefficient remains constant. This can be deduced from Fig. 67. The crystal was first extended at 4.2°, then cooled to 1.2° and further extended. On reheating to 4.2° the extension proceeded as before and the slope of the curve was constant throughout. In two cases, when

† M. Polanyi, E. Schmid, *Naturwiss.* 17 (1929), 301.

‡ W. Meissner, M. Polanyi, E. Schmid, *Z. Physik*, 66 (1930), 477.

the crystal was again extended after cooling from 20° to 12° and from 4.2° to 1.2° , there was a slight increase in hardness, about 10 per cent. The effect of straining at both temperatures is clearly shown in the diagram.

The results of these experiments may be summarized as follows. The yield-point does not change in value over a considerable range of temperature. It is independent of the rate of loading and is very much lower than that demanded by theory.[†] Beyond the yield-point the stress-strain curve shows a very great hardening of the metal. Within certain temperature limits this curve is independent of temperature and rate of deformation. The authors call this type of plasticity 'athermal'.

As opposed to this, they describe a 'thermal' plasticity which is dependent on temperature. They compare this to viscous flow and to the results obtained on testing amorphous substances. Here the stress depends on temperature and rate of testing.

A combination of both types of plasticity occurs in most metals tested at intermediate temperatures.

The effect of low temperatures on the tensile properties of iron and steel has been investigated by Sauerwald[‡] and his collaborators. Single crystals became suddenly brittle at -155°C . Comparing large crystals with polycrystalline material, an inter-crystalline fracture was found to occur between -144° and -154°C . There is no explanation of these phenomena, although the workers quoted attribute them to variation in cohesion with temperature along different crystal directions. When fracture occurs at these low temperatures the cleavage plane is parallel to $\{100\}$ faces of the crystal.

It is reported[§] that the rigidity of tin and mercury is practically unaltered when the metal passes from the ordinary to the super-conductive state, although a tensile stress affects the temperature at which the change occurs.||

These tests may be compared with some actually carried out with amorphous substances in which the rate of flow is proportional to the

[†] See Chapter XV.

[‡] F. Sauerwald, K. A. Pohle, *Z. Physik*, **56** (1929), 576. F. Sauerwald, B. Schmidt, H. Diententhal, *Z. Physik*, **61** (1930), 153. F. Sauerwald, B. Schmidt, G. Kramer, *Z. Physik*, **67** (1931), 179.

[§] W. J. de Haas, M. Kinoshita, *Comm. Phys. Lab. Univ. Leiden*, No. 187 (1928), 17.

|| F. Sauerwald, K. A. Pohle, *Z. Physik*, **56** (1929), 576. F. Sauerwald, B. Schmidt, H. Diententhal, *Z. Physik*, **61** (1930), 153. F. Sauerwald, B. Schmidt, G. Kramer, *Z. Physik*, **67** (1931), 179.

temperature.† The properties of these substances will be referred to again in the next chapter.

2. Effect of temperature and duration of test on polycrystalline metal

From the foregoing tests on single crystals, it will be seen that as the temperature rises the yield-point is lowered and also the rate of hardening. The total amount of possible elongation before fracture is greater. This is also true for crystal aggregates.‡ In Fig. 68 are ex-

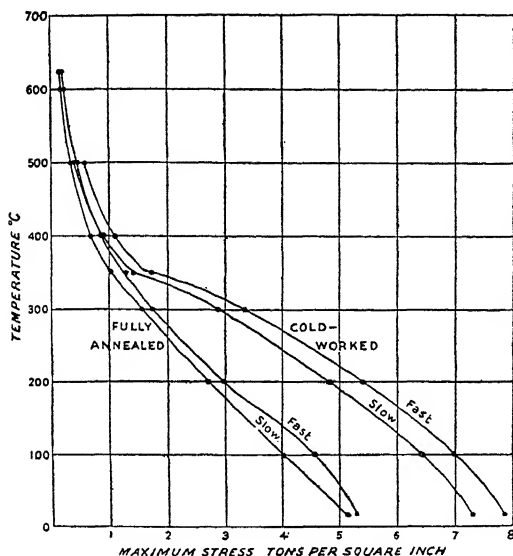


FIG. 68. Tests on aluminium at different temperatures.

amples of the results obtained with aluminium.§ Similar results have been found in copper (Fig. 69).|| The somewhat rapid changes that take place between 300° and 400° C. are probably connected with the fact that cold-worked aluminium shows a great increase in the rate of recrystallization at about 350° C.†† The change in direction of the curves is particularly marked in those obtained from cold-worked specimens.

† R. Becker, *Z. techn. Physik*, 7 (1926), 547. W. Braunbek, *Z. Physik*, 57 (1929), 501.

‡ Certain alloys, especially castings, show an increased strength with rise of temperature. This must be attributed to increased solubility of another constituent.

§ T. Martin, *J. Inst. Metals*, 31 (1924), 121. For copper see next footnote.

|| G. D. Bengough, D. Hanson, *J. Inst. Metals*, 12 (1914), 56.

†† H. C. H. Carpenter, C. F. Elam, *J. Inst. Metals*, 25 (1921), 259.

Different amounts of cold work prior to testing† also affect the results. As soon as a temperature is reached at which recrystallization takes place, the strength of all the test-pieces approaches the same value. In such tests the duration of the test becomes increasingly important.

An experiment carried out by Schmid and Wassermann‡ was intended to show the effect of time at different temperatures on the extension of cold-worked copper wires. They found that the wires

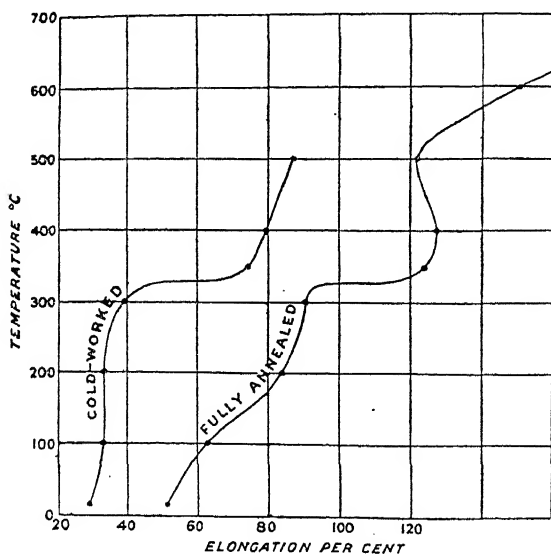


FIG. 69. Tests on copper at different temperatures.

extended very rapidly at a temperature corresponding to that of rapid recrystallization (Fig. 70).

A very large number of metals have been tested at different temperatures with results similar to those quoted. References to some of them are given below.§ All methods of deformation are similarly affected by temperature and many commercial processes are carried out at high temperatures in order to take advantage of the easier working and greater ductility.

† D. H. Ingall, *J. Inst. Metals*, **30** (1923), 171.

‡ E. Schmid, G. Wassermann, *Z. Metallkunde*, **23** (1931), 242.

§ P. Ludwik, *Z. Verein. deuts. Ing.* **59** (1915), 657. *Intern. Z. Metallog.* **8** (1916), 53.
G. Sachs, *Grundbegriffe der Mech. Techn. der Metalle*. W. P. Sykes, *Trans. Amer. Inst. Min. Met. Eng.* **60** (1921), 780. Huntington, *J. Inst. Metals*, **8** (1912), 126.
D. Hanson, M. A. Wheeler, *J. Inst. Metals*, **49** (1931), 229.

The commercial importance of iron and steel at high temperatures has resulted in a large number of tests being carried out with this

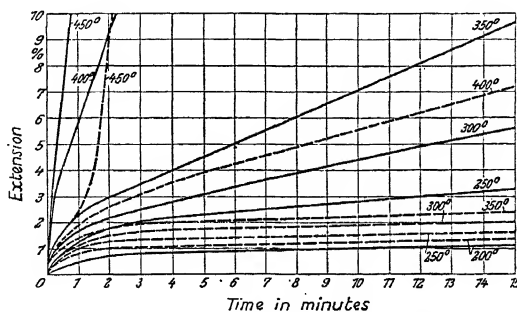


FIG. 70. The effect of temperature on rate of extension of copper wires under constant load.
— hard --- soft.

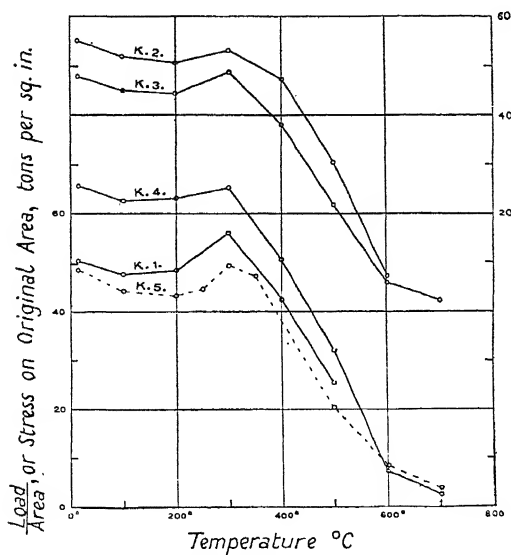


FIG. 71. Alloy and carbon steels tested at high temperatures.

metal. Fig. 71 shows the results of tensile tests on a number of steels from 0° to 600°C .†

Rosenhain and Humfrey‡ measured the tenacity and deformation

† F. C. Lea, *Proc. Inst. Mech. Eng.*, 1924, 1051.

‡ W. Rosenhain, J. C. W. Humfrey, *J. Iron Steel Inst.* 87 (1913), 219.

of mild steel up to temperatures above 1000°C . Certain well-defined breaks in the curves correspond with the well-known recalescence points of iron and steel, one of which is associated with an allotropic change at 900°C . A comparison of the rate of testing at two different temperatures was made. The faster the rate of extension the higher the tensile stress.

3. Long-period tests at high temperatures

The very great influence of rate of testing at temperatures above normal has led to a large number of researches on the deformation of metals over long periods, under loads well below their normal breaking loads. This has resulted partly from a demand for materials that will withstand high working stresses at comparatively high temperatures. It is well known that a metal will slowly deform ('creep') under the prolonged application of a load which, if applied momentarily, will produce no measurable plastic deformation. Hopkinson,[†] in fact, showed that if a load exceeding that required to break a copper wire in a tensile static test, was applied for $1/1000$ second, there was no plastic extension and there is evidence that if the frequency in alternating stress tests is sufficiently high the metal is unaffected.[‡]

Metals of low melting-point will 'creep' at normal temperatures. For example, the flow of lead roofing is well known. Andrade[§] measured the rate of flow of lead, tin, copper, iron and solid mercury at different temperatures and found that it could be expressed by means of an empirical formula and that the flow was of a viscous nature, i.e. the metal continued to extend at a uniform rate under a given stress. By an ingenious method he maintained a constant stress so that his results are comparable with those on single crystals.|| Like these, his curves show two stages. Only after a certain amount of deformation has taken place are the metals truly viscous, except at high temperatures.

Careful measurements at different temperatures and at very slow rates of extension have shown that the deformation of both single crystals and polycrystalline metals is discontinuous. It was originally

[†] B. Hopkinson, *Proc. Roy. Soc. A*, **74** (1905).

[‡] C. F. Jenkin, *Proc. Roy. Soc. A*, **109** (1925), 119.

[§] E. N. da C. Andrade, *Proc. Roy. Soc. A*, **90** (1914), 329.

|| See also J. Cournot and K. Sasagawa, *Comptes Rendus*, **181** (1925), 661.

demonstrated in the case of rock-salt by Joffé,[†] and the subject was further investigated by Classen-Nekludowa[‡] for rock-salt, α brass (both single crystals and microcrystalline material), and aluminium. The metals deform by a series of sudden extensions followed by periods of rest in which each extension may represent an increase in length of 5 to 6 μ or as much as 160 to 200 μ . Schmid and Valouch[§] have found a similar occurrence in very pure zinc crystals. This phenomenon is most marked over a certain temperature range—for example, from 230° to 500° C. for rock-salt, 450° to 550° C. for aluminium, and 410° to 550° C. for α brass.

If the temperature is too low the discontinuities are so small as to be hardly noticeable. If it is too high they follow one another with such rapidity that they are inseparable. In rock-salt the magnitude of the jumps depends on the temperature, but it is constant and independent of the temperature in the case of brass. The interval between successive jumps is variable but tends to decrease with increasing temperature. Their magnitude increases with the hardening, and their frequency appears to be a function of the difference between the imposed stress and the elastic limit at the moment of each successive jump. In the case of the zinc crystals, microscopic examination showed that each jump was associated with slip on the basal plane, and not with the formation of twins. The angle which the basal plane makes with the direction of pull (χ) affects the readiness with which this deformation takes place, i.e. it is almost absent when χ is 26–39° and is most marked at angles approximating to 60°. In zinc, the jumps are usually accompanied by a sudden drop in stress. Occurrences of this kind may be responsible for the ‘cry’ of tin, zinc, and certain other metals.

Somewhat similar experiments to these have been carried out by Hanson and Wheeler^{||} on aluminium, in both the single crystal and polycrystalline state. In addition to tests carried out at temperatures up to 400° C., they also tested the metals under conditions of very slow loading. In one instance the test-piece took 575 days to fracture. Each addition to the load caused a rise in the stress-strain diagram followed by a horizontal portion where no hardening took

[†] A. Joffé, *Physics of Crystals*. See also J. W. Obreimow and L. W. Schubnikoff, *Z. Physik*, **41** (1927), 907.

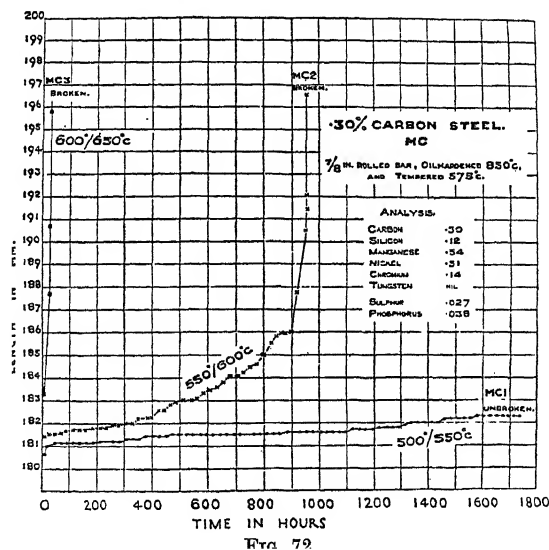
[‡] M. Classen-Nekludowa, *Z. Physik*, **55** (1929), 555.

[§] E. Schmid, N. A. Valouch, *Z. Physik*, **75** (1932), 531.

^{||} D. Hanson, M. A. Wheeler, *J. Inst. Metals*, **45** (1931), 229.

place, but the metal continued to extend. In the early stages of the test this 'creep' was small, but it became relatively larger as the test proceeded, and towards the end of the test addition to the load produced no further rise in the curve.

The time factor and the temperature at which such tests are carried out are clearly very important, but under the conditions of this experiment a point is reached where no further work-hardening takes place and there is no further absorption of internal energy. One can assume that the metal partially recrystallizes as it extends.



Most technical tests are carried out under constant load. This is not of very great importance while the extension is small, but as soon as the test-piece decreases in cross-sectional area the whole character of the test is changed, i.e. the metal is actually being subjected to increasing stresses. Allowing for this, the experiments that have been carried out in this manner are very interesting in showing the effect of time, when conditions are such that plastic yielding is going on slowly.

Fig. 72 is taken from a paper of Dickenson.† He is of the opinion that creep will proceed at all temperatures under a given load, given sufficient time.

† J. H. S. Dickenson, *J. Iron Steel Inst.* **106** (1922), 103.

Lea,[†] on the other hand, maintains that for every temperature there is a stress at which the work-hardening effect of the deformation is exactly equal to the annealing effect at that temperature.[‡]

4. Tests at temperatures near the melting-point

The normal fracture of metals takes place across the crystals rather than between them. This was illustrated in Chapter V, where the influence of the crystal boundary was considered. Near the melting-point, however, the crystals tend to pull apart and fracture becomes

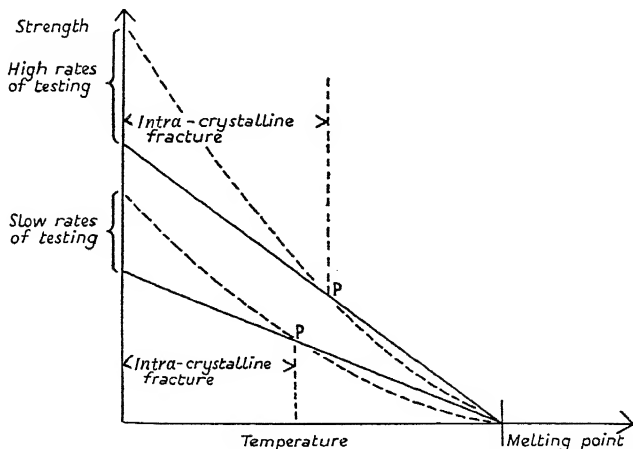


FIG. 73. Diagram illustrating relation between amorphous and crystalline metal when extended at different temperatures.

P = equicohesive temperature.

--- amorphous.

— crystalline.

inter-crystalline. This was shown to be the case for iron and steel by Rosenhain and Humfrey,[§] and for soft metals such as lead and tin, by Rosenhain and Ewen.^{||} Even in the case of soft metals, deformation of the individual crystals was almost entirely absent. This fact has been regarded as evidence in favour of an amorphous inter-crystalline layer, which is stronger than the crystalline matter at low temperatures, and weaker at higher temperatures. The relation between the two forms may be illustrated by means of Fig. 73 based

[†] F. C. Lea, *Proc. Inst. Mech. Eng.* (1924), 1053.

[‡] It is impossible to deal with the enormous amount of work that has recently appeared in connexion with 'creep'.

[§] W. Rosenhain, J. C. W. Humfrey, *J. Iron Steel Inst.* **87** (1913), 219.

^{||} W. Rosenhain, D. Ewen, *J. Inst. Metals*, **10** (1913), 119; **8** (1912), 149. See also G. D. Bengough, *J. Inst. Metals*, **8** (1912), 126.

on the work of Jeffries.† Jeffries represents the amorphous substance as decreasing in strength more rapidly than the crystalline and at some given temperature they may be regarded as having identical cohesion. This, Jeffries calls the 'equicohesive temperature'. At the melting-point the crystalline material may have a definite cohesion, whereas the amorphous has none. In further evidence of this hypothesis, Jeffries compared the strength of metal wires in a fine-grained and coarse-grained state at different temperatures. He found that below a certain temperature the fine-grained metal was the stronger, whereas above this temperature the coarse-grained specimen was stronger. The temperature varied for each metal tested, and corresponded roughly with their respective recrystallization temperatures. This he again attributed to a larger amount of intercrystalline amorphous substance in the fine-grained metal.

5. Effect of high rates of deformation

All the examples of tests carried out at different rates of deformation described in the last few sections show that the greater the rate of deformation, the higher the stresses produced. When, however, one comes to consider very high rates of fracture the results are inconclusive. Some early experiments of Maitland‡ on gun-steel showed that the ductility, as measured by elongation and reduction of area, was raised from 27 per cent. to 47 or even 60 per cent. if the steel was fractured by a charge of gun-cotton. Hopkinson§ showed that a mild-steel plate could be shattered by means of a charge of gun-cotton in close contact with it, although the individual pieces were undeformed. The influence of velocity of impact in impact tests is very marked. An extreme example quoted by Hadfield and Main|| as the result of a test in a Frémont Machine is as follows:

Mild Steel	Kg./mm. ²	Angle through which test-piece is bent
Slow	38.5	131°
Fast	1	1°

The effect of the rate of deformation in metals has been compared with the results obtained for amorphous substances, such as glass,

† Z. Jeffries, R. S. Archer, *Science of Metals*, p. 166.

‡ Maitland, *Proc. Inst. Civil Eng.* **89** (1886), 120.

§ B. Hopkinson, *Phil. Trans. Roy. Soc. A*, **213**, 437.

|| R. A. Hadfield, S. A. Main, *Proc. Inst. Civil Eng.* **21** (1920), 127.

pitch, certain waxes, etc. These substances are brittle when a force is suddenly applied to them, but will deform to almost any extent if the load is applied slowly. A substance like pitch will flow under its own weight exactly like a liquid, but can be broken with a sharp blow of a hammer. These substances flow much more readily at high

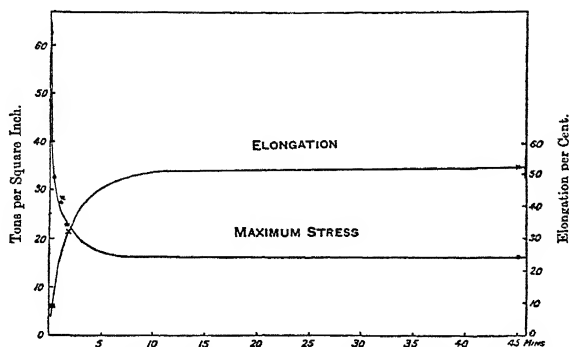


FIG. 74. Effect of rate of loading on mechanical properties of 4Z7.

temperatures, but have no distinct melting-point such as a true solid possesses. Metals possess certain of the properties of amorphous substances to different degrees according to the state in which they are tested. An alloy of zinc containing 4 per cent. copper and 7 per cent. aluminium has been described by Rosenhain, Haughton, and Bingham† which has many of the properties of an amorphous solid (Fig. 74).

† W. Rosenhain, J. L. Haughton, K. E. Bingham, *J. Inst. Metals*, **23** (1920), 261.

ALTERATION IN THE X-RAY REFLECTIONS ACCOMPANYING DEFORMATION†

THE X-ray reflections from worked metals differ in several respects from those of undeformed. The following changes might be expected:

1. Changes in the spacings of the crystal planes.
2. Changes in the character of the reflections. Under this heading one may include (a) elongation of the Laue spots. (b) Spreading of the angle over which characteristic X-rays are reflected, and an increase in the size of the reflected spots on a photographic plate. Figs. 1 and 2, Plate IV.
3. Broadening of the Debye-Scherrer rings; with which may be associated loss of resolution of the K_α doublet. Fig. 7, Plate IV.
4. Changes of intensity of reflection, which may affect all the reflections or only some of them.

1. The earliest investigations of severely worked metal wires and foils showed that there was no difference in the spacing of the lines obtained from powder photographs.‡ This leads to the conclusion that, whatever the distortion in the region of the slip-planes, there are relatively large layers or blocks of crystalline matter which are unaffected by the general deformation, except to a degree comparable with elastic distortion (see below). In a single crystal the spots on a photographic plate shift in accordance with extension within the elastic range, and by measuring the movement of the spots the elastic constants of the material can be calculated.§

2. As soon as the elastic limit is exceeded the character of the X-ray reflections changes entirely. Czochralski|| first noticed this phenomenon in metals and Joffé†† in rock-salt. In fact, Joffé and his collaborators were able to determine the elastic limit accurately by noticing the point at which the Laue spots began to elongate. The change in the reflection of characteristic X-rays is of a similar nature.

† A theoretical discussion of the observations described in this chapter is reserved until Chapter XV.

‡ M. Ettisch, M. Polanyi, K. Weissenberg, *Z. Physik*, **7** (1921), 181.

§ G. Sachs, J. Weerts, *Z. Physik*, **64** (1930), 344. A. Joffé, N. W. Kirpitschewa, N. A. Lewitzky, *Z. Physik*, **22** (1924), 286.

|| J. Czochralski, *Z. Metallkunde*, **15** (1923), 60.

†† Loc. cit.

If the crystal is perfect, there is a very sharp reflection spot on the photographic plate, and the setting angle at which reflections occur is confined to a small range. Even if a crystal is deformed in the most uniform manner possible, the spots increase in size and occur over a wider setting angle. This was noticed in the earliest experiments with zinc crystals,[†] and was thought to be due to an elastic bending of the slip-plane ('Biegegleitung').[‡] Joffé also interpreted the change observed in rock-salt as due to bending of the slip-plane about an axis corresponding to the direction of slip. He further showed that a cleavage flake of a distorted rock-salt crystal was so bent, but that the bending was permanent, i.e. plastic, not elastic.

Goucher§ carried the investigation farther in the case of distorted tungsten crystals, and found that the reflected spot from the slip-plane was longer if the section of the specimen was mounted so that the major axis of the ellipse intersected the spectrometer axis, than if the minor axis was in this position. Such a result might be obtained if an undistorted crystal were gradually inclined to the X-ray beam through an angle of 10° in one direction, and remained stationary when a similar photograph was taken at 90° to this direction. In other words, the crystal was broken up into fragments which were arranged on a curved surface, while remaining in alignment in a transverse direction.

Experiments with aluminium crystals were made by Taylor|| and Yamaguchi,^{††} who showed that the crystal was broken up on the slip-plane in a peculiar manner. Taylor's experiments were as follows. If a section was cut parallel to the slip-plane and mounted in a spectrometer so that the direction of slip was in the axis of rotation, the X-ray reflections from the slip-plane {111} were different in character from similar ones obtained when the specimen was turned through 90°, i.e. with the direction transverse to the direction of slip in the axis of rotation. In the former, the spots were elongated, but reflection only took place over a range of 2°. In the latter, the spots were small, but reflection took place over a range of 10° to 12°. Further, the maximum intensity occurred nearer one end of the range. This proves that the crystal particles have rotated in one direction transverse to the direction of slip, and may be compared to rollers between

[†] H. Mark, M. Polanyi, E. Schmid, *Z. Physik*, **12** (1923), 58.

[‡] M. Polanyi, *Z. Krist.* **61** (1925), 49.

§ F. S. Goucher, *Phil. Mag.* (2) (1924), 800; (2) (1926), 289.

|| G. I. Taylor, *Trans. Faraday Soc.* **24** (1928), 121.

^{††} K. Yamaguchi, *Sci. Papers Inst. Phys. Chem. Res. Tokyo*, **11** (1929), 151.

the slip-plane, although the rotation is much less than would be necessary if they actually functioned as rollers.

Yamaguchi† does not entirely agree with this interpretation, but by means of Laue photographs of aluminium crystals he confirmed the direction of curvature of the crystal. He further showed that the amount of rotation increased with the amount of shear for each crystal and was greater for hard crystals than soft. This is particularly interesting in showing that there is a close relationship between work-hardening and internal strain. It is also in agreement with the results obtained on testing crystals at high temperatures.‡ In this case, there is no increase in resistance to shear nor any appreciable amount of internal strain.

Burgers§ investigated the rotation of fragments on the slip-plane from the point of view of the orientation of the crystals produced by annealing and their relation to the worked structure. He found that the new crystals were related to the deformation in the same sense as the fragments causing elongation of the Laue spots are related to the orientation of the original crystal. But, whereas the distortion of the X-ray spots has been estimated as being caused by a rotation of from 10° to 20° , the new crystals are rotated a further 20° to 60° in regard to the position in the worked unannealed crystal. Fig. 92, p. 170, represents diagrammatically in two dimensions the nature of this supposed rotation.

The type of fragmentation suggested by Taylor and Yamaguchi has a further bearing on the problem. It is possible to imagine very great concentrations of stress locally while the total shear-stress distributed throughout the whole volume of the crystal is very much less. The concentration of stress would normally be at the ends of the slip-planes and would be of the same character as those at the ends of cracks or scratches on the surface. The influence of small discontinuities in causing very large stress concentrations is well known. This fact will be referred to again later. Taylor's explanation of his results was that if the separation of the crystal at the slip-plane were regarded as a crack, the ends of which were closed, the stresses in the ellipse so formed were concentrated at the ends of the ellipse and tended to produce an elastic rotation of the particles in a direction

† K. Yamaguchi, *Sci. Papers Inst. Phys. Chem. Res. Tokyo*, **11** (1929), 151.

‡ K. Yamaguchi, S. Togino, *ibid.* **9** (1929), 277.

§ W. G. Burgers, *Z. Physik*, **68** (1930), 605.

transverse to the direction of slip. Starr† confirmed this hypothesis by mathematical analysis, except on one point. The concentration of rotation did not coincide with regions of stress concentration.

3 and 4. Closely related to the change in the X-ray photographs described above is the broadening of the lines in a Debye-Scherrer spectrum.

Broadening of the lines also affects resolution of the K_α doublet and changes of intensity. These phenomena were first observed in worked metals by Davey‡ and van Arkel.§ Broadening of the lines occurs in most metals with the exception of aluminium|| and zinc.††

It has further been shown by a number of investigators‡‡ that by heating at a temperature below the recrystallization temperature the resolution of the K_α doublet of tungsten is regained. Goucher§§ showed that this was effected while the crystal fragments on the slip-plane still remained bent.||| von Göler and Sachs confirmed these observations, although separation of the lines was not really complete until after recrystallization had taken place.††† This is very difficult to explain if one tries to connect broadening of the lines with hardening.

Sekito††† has also shown the effect of heating on the width of the Debye-Scherrer lines obtained from cold-drawn copper wire. According to him, the internal strain is equivalent to 0.17 to 0.22 per cent. of the original lattice parameter. The maximum distortion is in the [111] and [100] directions lying in the wire axis. He calculates that this is equivalent to a stress intensity of 3.6×10^9 dynes/cm.², which is only a little less than the tensile strength of copper, which is about 4×10^9 dynes/cm.²

A number of measurements have recently been made by Wood.¶¶¶ He compared the incidence of lattice distortion, measured by broadening of the lines, with the amount of deformation in both drawn

† A. T. Starr, *Proc. Camb. Phil. Soc.* **24** (1927-8), 489.

‡ W. P. Davey, *Gen. Elec. Rev.* **28** (1925), 588.

§ A. E. van Arkel, *Physica*, **5** (1925), 208; *Naturwiss.* **13** (1925), 662.

|| U. Dehlinger, *Z. Krist.* **65** (1927), 615. W. G. Burgers, *Z. Physik*, **59** (1930), 651.

†† U. Dehlinger, *Z. Krist.* **65** (1927), 615.

‡‡ K. Becker, *Z. Physik*, **42** (1927), 226. U. Dehlinger, *Z. Krist.* **65** (1927), 615.

A. E. van Arkel, W. G. Burgers, *Z. Physik*, **48** (1928), 690. von Göler, G. Sachs, *Z. Metallkunde*, 1927.

§§ F. S. Goucher, *Phil. Mag.* (2) (1926), 289.

||| See also Karnop, Sachs, *Z. Physik*, **41** (1927), 116; **42** (1927), 283.

††† von Göler, G. Sachs, *Z. Metallkunde*, 1927.

††† S. Sekito, *Sci. Rep. Tohoku Imp. Univ.* **16** (1927), 343.

¶¶¶ W. A. Wood, *Proc. Phys. Soc.* **44** (1932), 67; *Phil. Mag.* **14** (1932), 656.

wires and rolled foils, together with the relation of these phenomena to the formation of a fibre-structure. In both cases lattice distortion increases rapidly with deformation. A maximum is reached which is maintained at a constant value. Preferred orientation does not appear until the maximum distortion is reached. He also found that the resolution of the K_α doublet disappeared when constantan wires were loaded in tension,[†] but that the distortion was not uniform. The same photograph showed resolution in the case of a (331) line, but not in the (420) line.

Burgers[‡] examined the different zones in a cold-drawn wire in order to find if the distortion was uniform. He dissolved off successive layers of the metal and compared the broadening of the lines of a Debye-Scherrer photograph. He found the distortion to be greatest in the centre and outside skin of the wire. This result may be compared with that of Schmid and Wassermann§ on the texture of hard-drawn wires. They found the core to have the most perfect fibre-structure. The tensile strength is also greatest in a zone near the core.||

Intensity measurements by means of a photometer have been made on worked and annealed metals by Hengstenberg and Mark.^{††} Changes of intensity are related to broadening of the lines. There is no change of relative intensity with uniform elastic extension. Local distortion, such as slipping on a given plane, affects the relative intensity of the lines but does not produce broadening. Mark and Hengstenberg discuss the theoretical aspect of broadening, resolution of K_α , and change of intensity. By means of a polarization microscope they measured the change of intensity of an (800) plane in a potassium chloride crystal, both worked and unworked. The following table gives their results:

	<i>I</i>			
Undeformed crystal (800) plane	.	.	.	105.8%
4 per cent. compressed	.	.	.	98.7
Heated 10 hrs. at 400° C.	.	.	.	103.5
Control	.	.	.	100

[†] W. A. Wood, *Nature*, **129** (1932), 760. It has always been stated that loading in tension does not result in loss of resolution of the K_α doublet. See Dehlinger, Burgers, etc.

[‡] W. G. Burgers, *Z. Physik*, **58** (1929), 11.

[§] E. Schmid, G. Wassermann, Chapter V.

^{||} F. W. Harris, *Amer. Inst. Met. Div.* (1928), 518.

^{††} J. Hengstenberg, H. Mark, *Z. Physik*, **61** (1930), 435. For similar measurements on potassium chloride crystals by optical means, see R. Brill, *Z. Physik*, **61** (1930), 454.

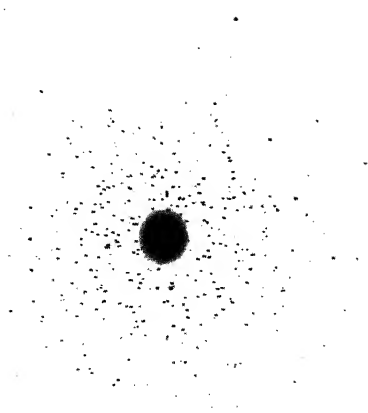


FIG. 1. Laue photograph through aluminium consisting of a number of large crystals

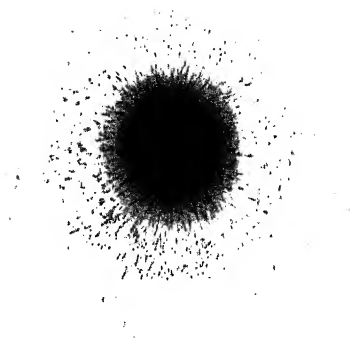


FIG. 2. Same as Fig. 1, after rolling, showing elongation of Laue spots and beginning of formation of Debye rings



FIG. 3. Debye rings in worked magnesium

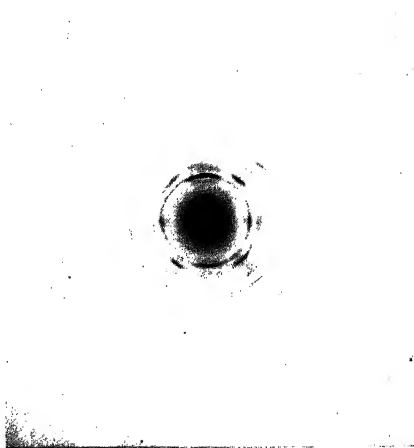


FIG. 4. Transmission photograph of rolled aluminium sheet, showing 'preferred' orientation

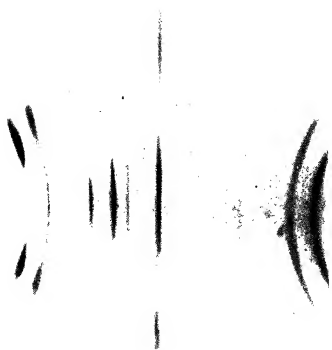


FIG. 5. Reflection photograph of rolled copper, showing 'preferred' orientation



FIG. 6. The same as Fig. 5, illustrating structure obtained when crystals of considerable size are orientated as in rolled state. This is typical of a partially annealed metal



(a)

FIG. 7. (211) line in tungsten steel. (a) Separation of K_1 doublet, no distortion. (b) Line diffuse, distortion

Calculations lead to the conclusion that with a compression of 3.6 per cent. the maximum shifting of the atoms along directions $[110]$ and $[\bar{1}\bar{1}0]$ is about $\frac{1}{8}$ distance in these directions. This corresponds to a change of energy of less than 1 per cent., which is much less than that found by heat-deformation measurements.

Although broadening of the lines, etc., usually accompanies work-hardening (and also age-hardening)[†] and consequently lattice distortion is frequently referred to as a cause of increased hardness, some recent experiments of Fink and van Horn[‡] have tended to show that under certain conditions lattice distortion is accompanied by softening. Strips of aluminium alloy and α brass were bent and the region under strain tested for hardness by an indentation method. There was a small decrease in hardness, while X-ray photographs showed a diffusion of the spots and loss of resolution of the K_{α} doublet. The subject was further investigated by Kokubo.[§] He showed that under a tensile stress, i.e. on the convex side of the bent specimen, penetration would be facilitated and the metal appear softer, while on the concave side, which is under compression, penetration is impeded and the metal would appear harder. This is confirmed by measurement. These experiments tend to show that the direction of the applied stress and the type of hardness measurement employed may have a considerable influence on the experimental results obtained.

An investigation of polished surfaces^{||} of metals has also been made by means of cathode rays. Somewhat similar phenomena are met with such as have been described above. The diffraction rings become diffuse and are sometimes lost, being replaced by others corresponding to a spacing differing from that characteristic of the metal. The conclusion is drawn that polished surfaces are largely amorphous and that the distances apart of the atoms in this layer are not the same as in normal crystals.

[†] M. L. V. Gayler, G. D. Preston, *J. Inst. Metals*, **41** (1929), 191.

[‡] W. L. Fink, K. R. van Horn, *J. Inst. Metals*, **44** (1930), 241.

[§] S. Kokubo, *Sci. Rep. Tohoku Imp. Univ.* **21** (1932), 256.

^{||} R. C. French, *Proc. Roy. Soc. A*, **140** (1932), 637.

XI

RELATION BETWEEN WORK DONE AND ENERGY ABSORBED

THAT heat is developed during plastic deformation is a well-known fact. A tensile test-piece of a hard metal is frequently too hot to hold in the hand after fracture. The laws governing the change in temperature of metals during mechanical straining were established by Lord Kelvin in a number of investigations. He established the following equation:

$$\theta = \frac{tep}{JK\rho},$$

where

θ = elevation of temperature produced by the sudden application of stress p ,

t = absolute temperature of substance,

e = geometrical effect (expansion or other strain) produced by a temperature of 1° when the body is kept under constant stress,

K = specific heat of the substance per unit mass under constant stress,

ρ = density,

J = Joule's equivalent.

Assuming perfect elasticity, and provided all thermal losses due to conduction, radiation, etc., are taken into account, cold is produced when a metal is strained by opposing, and heat when a metal is strained by yielding to any elastic force of its own. If a metal is strained in tension, there is first a cooling effect in the elastic range, followed by a sudden evolution of heat when it begins to deform plastically. This fact has been made use of, particularly in connexion with alternating stress tests, in the determination of the elastic limit.† While a metal is being tested within a region of perfect elasticity, there is no change of temperature. But, as soon as elastic hysteresis sets in, i.e. the stress-strain diagram becomes a loop, there is a heat evolution.

In view of possible explanations of the hardening of metals by

† B. Hopkinson, G. T. Williams, *Proc. Roy. Soc. A*, 87 (1912), 502. E. G. Coker, *Trans. Roy. Soc. Edin.* 41 (1904), 229.

plastic deformation, it is very important to compare the work done in producing a certain deformation and the heat developed in the metal by the process. The difference is a measure of the energy absorbed by the metal and is responsible for the change in physical and mechanical properties which are characteristic of worked metals. There are two methods of measuring this difference. 1. The difference between the heat equivalent of the work done, and the rise in temperature of the specimen, can be determined during a mechanical test. 2. The internal energy of a work-hardened metal may be determined by some direct form of measurement.

H. Hort† measured the quantity of heat produced by extending a soft iron rod in a calorimeter and comparing this with the work done as calculated from the area of the stress-strain curve. He found a loss of from 5 to 15 per cent. compared with the work done.

Taylor and Farren‡ used metal tubes with a thermocouple attached inside, and extended the tubes as rapidly as possible so as to reduce heat losses due to conduction, radiation, etc., to a minimum. The difference between the direct rise in temperature and the temperature rise equivalent to the work done for the same extension amounted to 13.5 per cent. in the case of a mild steel, 8 to 9.5 per cent. in the case of copper, 7 to 8 per cent. in aluminium, and 4.5 to 5 per cent in aluminium single crystals.

Cold-worked metals dissolve in acids more rapidly than annealed,§ and the difference in the heat of solution of several metals in both states has been measured by C. S. Smith.|| The increase in potential energy of the worked metal over the annealed per gramme of material was found to be 10.5 calories in the case of copper, 9.0 calories in that of brass, and 6.7 calories in the case of zinc. This method has the disadvantage that the heat of solution is very large compared with the energy of internal strain. The amount of work done in preparing his specimens was unknown, as they were in the form of cold-drawn wires.

Sato†† deformed a number of metals by torsion, keeping them cool as far as possible. The work done was calculated from the area of the torsion diagram, and the latent heat was measured calorimetrically or

† H. Hort, *Z. Verein. deuts. Ing.* 50 (1906), 1831.

‡ W. S. Farren, G. I. Taylor, *Proc. Roy. Soc. A*, 107 (1925), 422.

§ G. Tammann, *Lehrb. Metallog.* 138. (English translation, Chemical Catalogue Co., New York, 1925.) || C. S. Smith, *Proc. Roy. Soc. A*, 125 (1929), 619.

†† S. Sato, *Sci. Rep. Tohoku Imp. Univ.* 20 (1931), 140.

by a method of differential thermal analysis. If the potential difference between a worked and an annealed specimen was measured and plotted against rise of temperature over a certain range, Sato found that, at a certain temperature, the potential difference indicated a heat evolution corresponding to a very considerable release of energy. This corresponded in many cases to the beginning of the recrystallization range for the particular metal. His results do not agree with Taylor and Farren's. Likewise, 1.08 gm. calories found for brass by Sato is very different from 9.0 gm. calories found by Smith.

Masima and Sachs† studied the heating effects of small deformations (1–4 per cent.) on brass crystals. The rise in temperature was measured by means of thermocouples. The heat evolved increased more rapidly as the extension proceeded, and the amount of heat produced exceeded by 20–30 per cent. that which would be expected from the extension of the test-piece. The authors suggest that more heat energy is produced than can be attributed to transformation of mechanical energy, and that the internal energy of the crystal actually decreases as the deformation progresses. This may be connected with the peculiarities in the deformation of brass crystals, in which there is a sudden change of slip-plane, accompanied by a sudden development of heat.

A recent attempt has been made by Rosenhain‡ and Stott to measure the heat liberated during the process of wire drawing and to compare this with the work done in drawing the wire. The experimental difficulties were considerable. It was found to be very difficult to measure the work done in wire drawing, and although the total amount of heat to be measured can be regulated by the amount of metal drawn, the rate of evolution of heat is slow and the corrections to be made in the calculations to allow for heat losses become relatively large. The energy absorbed by the metal was equivalent to 1.2 per cent. of the total work done (including friction of the die) in case of aluminium, 3.1 per cent. for annealed copper, and 0.5 per cent. for hand-drawn copper.

Taylor and Quinney§ have also measured the heat evolved in relation to the work done in torsion. The direct rise in temperature was measured by means of a thermocouple, and the specimen was also

† M. Masima, G. Sachs, *Z. Physik*, **56** (1929), 394.

‡ W. Rosenhain, V. H. Stott, *Proc. Roy. Soc. A*, **140** (1933), 9.

§ G. I. Taylor, H. Quinney, *Proc. Roy. Soc. A*, **143** (1934), 307.

quickly removed from the machine and dropped into a calorimeter. The results were in fair agreement. The proportion of heat retained by the metal decreases as the amount of work done increases. Saturation was probably not reached, owing to failure of the specimen, but was found to be of the order of 14 calories per gramme at 15° C.

By using compression specimens instead of torsion, a very much larger amount of work can be done. When the work done was equivalent to 15 calories per gramme, no further rise in compressive stress occurred with more compression. This suggests a close relation between latent energy and increase of strength due to mechanical deformation.

The results described above cannot be considered to be entirely satisfactory. The experimental difficulties are great in every method of attack. All that can be said at present is, that evidence is in favour of a retention by the metal of some of the energy imparted by the work done, but the magnitude of the latent energy can only be roughly estimated.

XII

CHANGES IN PHYSICAL PROPERTIES CAUSED BY DEFORMATION

1. Changes in elastic properties

The changes in the elastic properties have already been described, particularly in reference to the effects of repeated stresses. They may be summarized very briefly as follows:

(a) One can distinguish two forms of the stress-strain curves of metals in the elastic range, i.e. in the range when removal of the load results in a return of the metal to its original dimensions. (1) The stress may be proportional to the strain and give a straight line relationship and obey Hooke's Law. (2) The stress is not proportional to the strain.

(b) The elastic limit in tension or compression is raised by tension or compression towards further stresses acting in the same direction.

(c) The elastic limit is lowered by tension towards compressive stresses and vice versa (Bauschinger effect).

(d) Metals which are strained beyond the elastic limit and are immediately retested are frequently found to behave as non-elastic bodies, i.e. the stress-strain curve is not a straight line. This property is restored by resting or heat-treatment.

(e) A certain amount of elastic strain takes place with time after the maximum load has been applied. Similarly a release of strain (after-working) takes place after removal of the load.

The amount of movement may be considerable after severe working, particularly on the application of heat.† Although some of these experiments hardly come within the province of 'plastic deformation' they are very closely allied to it and will be briefly described as they must be taken into consideration when discussing the theory of deformation.

V. Wartenberg twisted specimens and measured the angle through which the metal moved in the reverse direction when the force was removed. One of his most interesting discoveries was that single crystals of both zinc and tungsten showed no elastic after-working, whereas the polycrystalline form of these metals did.‡

† V. Wartenberg, *Verh. deuts. phys. Ges.* **20** (1918), 113.

‡ This is not entirely in agreement with the work of Gough, Hanson, and Wright, which is described in Chapter VII.

It is also interesting to record that after-working does not take place at temperatures of liquid air† and, as V. Wartenberg pointed out, it is more marked in soft metals than hard.

Polanyi and Sachs‡ measured the untwisting of an iron tube on heating which had previously been subjected to torsion. They were thus measuring not the amount of elastic distortion which caused the metal to untwist directly the load was removed but the stored elastic energy which would only be released on heating. The rate at which untwisting took place was also measured and it was found to be much more rapid at certain temperatures, depending on the previous degree of distortion.

These experiments have been briefly referred to as they bring out the interdependence of elastic and plastic deformation. Instead of weakening a metal towards elastic deformation, plastic strain actually enables it to resist greater elastic strains and to deform elastically a greater amount.

Young's Modulus of Elasticity is altered by working, although to very different extents. The following table is taken from Tammann's *Lehrbuch der Metallographie* and is based on the work of Wertheim:§

Metal	Hard Kg./mm. ²	Soft Kg./mm. ²	Increase per cent.
Cu	12·449	10·519	18·3
Ag	7·357	7·140	3·0
Au	8·31	5·584	45·6
Pb	1·883	1·727	9·0
Fe	20·869	20·794	0·3

Goerens|| confirmed that there is very little change in the case of iron and steel.

According to Honda and Yamada†† the modulus of annealed steel is decreased by stretching, while in the case of non-ferrous metals such as copper, aluminium, and brass, it is raised. This is confirmed by other workers.‡‡

In order to investigate this anomaly, Kawai§§ carried out experi-

† A. Schafer, *Ann. Physik*, **5** (1901) 222.

‡ M. Polanyi, G. Sachs, *Z. Metallkunde*, **17** (1925), 227. See also E. Schmid, G. Wassermann, *Z. Physik*, **40** (1926), 451.

§ Wertheim, *Pogg. Ann.* **78** (1849), 391.

|| P. Goerens, *J. Iron Steel Inst. Carnegie Schol. Mem.* (1911), 320.

†† K. Honda, Yamada, *Sci. Rep. Tohoku Imp. Univ.* **17** (1928), 723.

‡‡ H. Brislee, *Trans. Faraday Soc.* **9-10** (1913), 162. W. Kuntze, *Z. Metallkunde*, **20** (1928), 145.

§§ T. Kawai, *Sci. Rep. Tohoku Imp. Univ.* **19** (1930), 209.

ments on both ferrous and non-ferrous metals. He found for iron and steel that the modulus of elasticity was lowered by stretching and drawing, and that it reached an almost constant value. Recovery was partial with low temperature annealings and more rapid as 900° was approached.

For non-ferrous metals Young's modulus first decreases rapidly, and then begins to increase slowly as the extension proceeds.

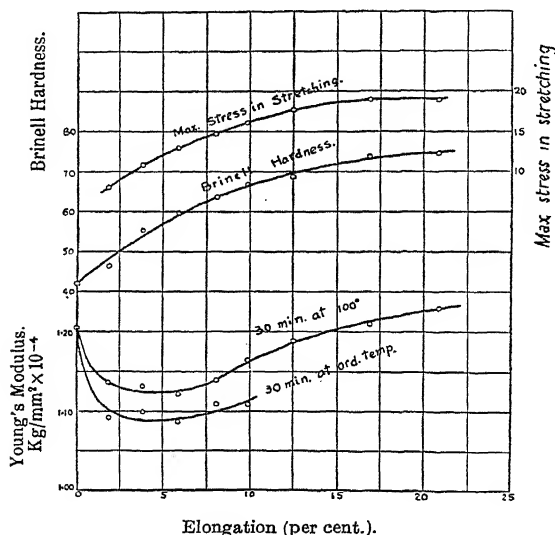


FIG. 75. Tested after 30 min. at ordinary temperature and 100° C.

The change in the modulus of rigidity due to cold work has been measured by a number of workers.† There is first a decrease for all metals. In copper it decreases until a minimum is reached and then remains constant. In nickel it decreases rapidly and then slowly. In iron and steel it decreases at first, reaches a minimum, and then increases.

Kawai† has attempted to attribute some of these phenomena to the effect of three factors; (1) internal stress; (2) rotation of crystals so that certain directions lie in the axis of the wire; (3) refining of crystal size. With regard to (2), the elastic properties of crystals do

† T. Kawai, *Sci. Rep. Tohoku Imp. Univ.* **20** (1931), 681. See also references to other work in this paper.

vary in different directions although these variations are comparatively small in crystals of cubic symmetry.†

The reverse changes take place when worked metals are heated. There is a rise in the value of both Young's modulus and the modulus of rigidity followed by a fall to the normal values when recrystallization takes place.‡

2. Changes in density

The effect of working a metal is usually to increase the density in the first instance by closing up pores and cracks. This is particularly noticeable in castings where flaws and blow-holes may be present to a considerable extent. With severe working, however, there is usually a small, but quite definite decrease in density.§ That this is definitely related to the deformation can be shown by annealing the specimen, when it will be found that the density rises again. In fact, most of the measurements of density changes have been made in this way. The decrease in density is attributed to the formation of amorphous metal on the planes of slip. This would have a lower density than metal in the crystalline state.||

O'Neill†† carried out some density measurements on a test-piece of armco iron. The test-piece was broken and cut into sections and the density of each section determined. The results indicate a decrease in density which increases rapidly near the point of fracture.

Some measurements carried out by the same author with a small iron crystal containing 1.81 per cent. silicon showed no change of density, but this result is doubtful.

Tests on brass crystals‡‡ show a rather sudden drop in density associated with the points in the course of extension at which double slipping should begin, i.e. the symmetrical inclination of two octahedral planes to the axis. Double slipping does not take place in brass crystals, or only to a limited extent. Information on this point will be found in the section on alloy crystals. It is surprising to find the density following the orientation rather than the load-extension

† Voigt, *Lehrbuch der Kristallphysik*.

‡ T. Kawai, loc. cit.

§ W. E. Alkins, *J. Inst. Metals*, **8** (1920), 381. G. Kahlbaum, *Z. anorg. Chem.* **46** (1905), 217.

|| G. Beilby, *Aggregation and Flow of Solids* (Macmillan).

†† H. O'Neill, *J. Iron Steel Inst.* **109** (1924), 93.

‡‡ M. Masima, G. Sachs, *Z. Physik*, **54** (1929), 666. See also T. Ueda, *Sci. Rep. Tohoku Imp. Univ.* **19** (1930), 473.

curves (Fig. 76). The effect of annealing is to raise the density, but this does not take place uniformly.

3. Changes in electrical properties

The specific electrical resistance, the temperature coefficient of specific resistance, and the thermoelectric force are all altered by mechanical work. During the region of elastic deformation the resistance is usually increased by extension and decreased by contraction. Under hydrostatic pressure, a large number of metals have been tested by Bridgman.[†] In most cases there is a decreased resistance although

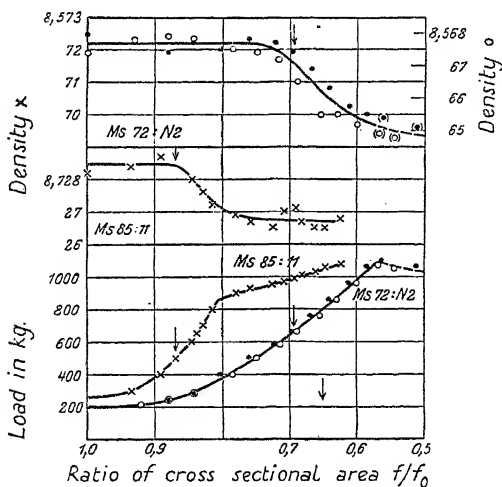


FIG. 76. Change in density of brass crystals with extension. \downarrow = point at which two slip-planes are symmetrically inclined to the axis.

there are some exceptions. The thermal coefficient of resistance is also altered, sometimes in one direction, sometimes in the other.

Changes accompanying plastic deformation, such as during wire drawing, have been measured by a number of workers, together with the reverse process of heating a hard-drawn wire and measuring the change of resistance accompanying the annealing process. Alkins[‡] observed the increase in electrical resistance with increase in strength in the case of copper wires.

[†] P. W. Bridgman, *Proc. Nat. Acad. Sci.* 6 (1920), 505. *Proc. Amer. Acad. Arts, Sci.* 57 (1922), 39, 75; 58 (1923), 163; 59 (1923), 107.

[‡] W. E. Alkins, *J. Inst. Metals*, 31 (1924), 157.

The effect of direct tension on the specific electrical resistance of steel, copper, and brass was measured by Ueda,[†] who at the same

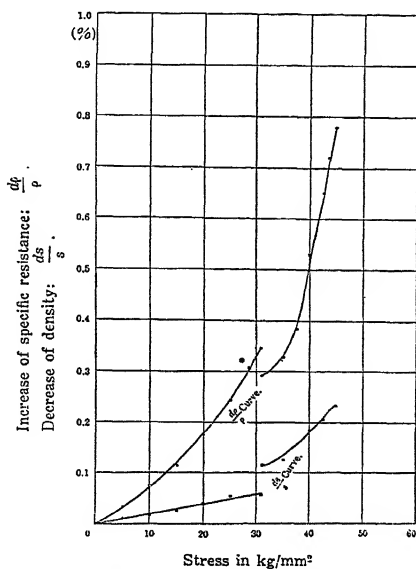


FIG. 77. 0.4 per cent. C Steel.

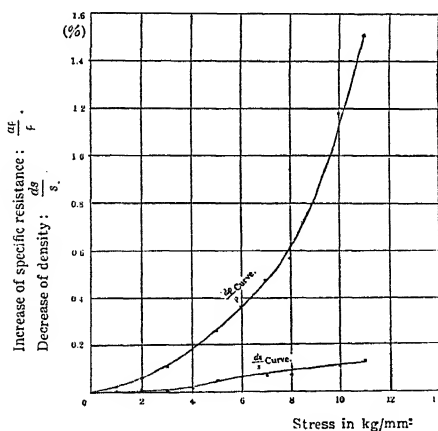


FIG. 78. Copper.

time compared the density (Figs. 77 and 78). In the case of iron and steel the resistance increases with tensile stress. There is a marked

[†] T. Ueda, *Sci. Rep. Tohoku Imp. Univ.* **19** (1930), 473.

discontinuity at the yield-point followed by a more rapid increase, which, however, tends to become less as the carbon content increases. The resistance increases similarly with copper and brass, but much more rapidly in the region of greater plastic deformation.

Lees and Calthrop† have investigated the effect of torsion on the conductivity and find a small decrease. Calthrop‡ extended his researches to single crystal wires with similar results.

The temperature coefficient of electrical resistance shows a decrease, but it is only possible to carry out measurements over a small range of temperatures so as not to destroy the effects of work-hardening. Geiss and van Liempt determined the values for tungsten wires and found that the coefficient decreased from $\alpha = 482 \times 10^{-5}$ to $\alpha = 290 \times 10^{-5}$ when the metal was drawn to a cross-section of 9.9μ .

Masima and Sachs§ measured the change in electrical conductivity in single α brass crystals when these were extended by pulling. There was a sudden increase in resistance at the beginning of the test and a more gradual but quite definite increase in the last stages of the test, but no very close relation between curves representing load and reduction in area and increase in specific resistance.

Bridgman|| measured the change in resistance when metal strips were subjected to tension both in the direction of the applied tension and transverse to it. There was usually an increase in resistance except in the case of nickel. In most cases the change was greater in the direction of the stress than transverse to it, and not necessarily of the same sign.

Other references

- W. Geiss, J. A. M. van Liempt, *Z. anorg. Chem.* **133** (1924), 107; **143** (1925), 259. *Z. Physik*, **41** (1927), 887.
E. Gruneisen, E. Goens, *Z. Physik*, **44** (1927), 638 (Table).
F. Credner, *Z. phys. Chem.* **82** (1913), 457.
G. Elsner, P. Siebe, *Z. Metallkunde*, **22** (1930), 397.
P. W. Bridgman, *Proc. Amer. Acad. Arts Sci.* **63** (1928), 329.

4. Changes in magnetic properties

As the magnetic properties of iron are so important, it is not surprising that these have been studied more fully than similar effects in other metals.

† C. H. Lees, J. E. Calthrop, *Proc. Phys. Soc.* **35** (1923), 225; **36** (1924), 168.

‡ J. E. Calthrop, *Proc. Phys. Soc.* **38** (1926), 207.

§ M. Masima, G. Sachs, *Z. Physik*, **51** (1928), 321.

|| P. W. Bridgman, *Proc. Amer. Acad. Arts Sci.* **59** (1923), 117; **60** (1925), 423.

The effect of an extension of 10 per cent. on the magnetization curve of an iron bar is shown in Fig. 79.† The saturation value, the susceptibility, the permeability and remanent magnetism are all lowered by the extension while the coercive force is raised. The hysteresis losses are also increased.‡ The effect of drawing iron wire was determined by Goerens.§ The change of properties is reversed by heating§|| (Fig. 80).

Fig. 82, which is due to Gerlach,†† compares the magnetization

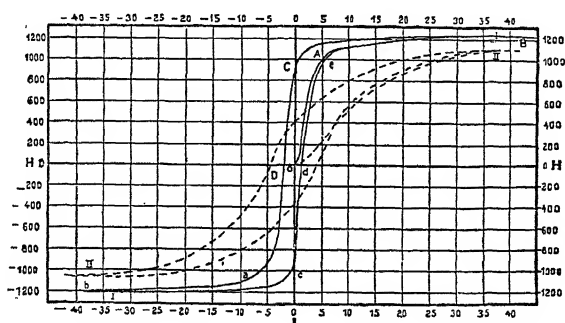


FIG. 79. Magnetization curves of iron before and after extension. — before. ---- after.

curves of iron in the form of a single crystal (I); after the crystal had been bent and straightened once (II); and after it had been bent backwards and forwards until it had become hard (III). The hysteresis is increased. He also showed that after a worked specimen had been heated the magnetic properties returned to normal. If the magnetic properties of single and polycrystalline iron are compared the chief difference is found to lie in the almost entire absence of hysteresis effects in the single crystal (Fig. 81).

Gerlach further investigated this problem with a view to correlating these differences with those observed on straining. It was then found that, if sufficient care was taken in the preparation of the iron, polycrystalline metal could be obtained which had the same low hysteresis losses. It may be concluded therefore that merely fineness of crystal

† G. Tammann, *Metallography*, Chemical Catalogue Co., New York, 1925.

‡ See also Honda and Terada, *Phil. Mag.* **14** (1907), 65.

§ P. Goerens, *J. Iron Steel Inst. Carnegie Schol. Mem.* (1911), 320.

|| E. Maurer, *Rev. Mét.* **5** (1908), 711.

†† W. Gerlach, *Z. Physik*, **38** (1926), 828; **39** (1926), 827. See also E. Gurnich, *Elektrotech. Z.* **36** (1915), Nos. 51 and 52; R. Forrer, *Comptes Rendus*, **180** (1925), 1253 and 1394.

grain is not responsible for the changes accompanying deformation, but that imperfections in the lattice produced by strain or due to other causes, are responsible.

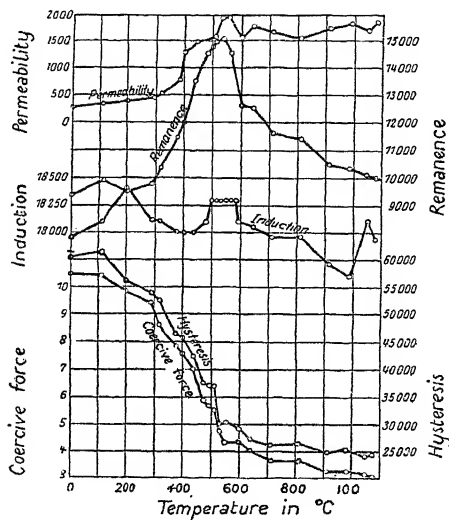


FIG. 80. Alteration of magnetic properties of hard-drawn iron (0.08 per cent. C) on heating.

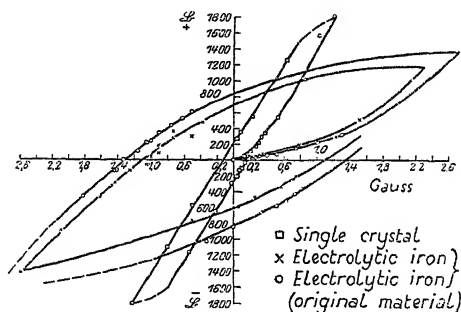


FIG. 81.

The influence of a tensile stress on the work done in hysteresis of nickel wires for different fields has been measured.† No break in the curves corresponding to the elastic limit was found, except for low fields. For low fields, a tensile load tends to increase the work done in hysteresis as it does also the magnetostriction and then to decrease

† R. Wwedensky, J. Simanow, *Z. Physik*, 38 (1926), 202.

it, and the change corresponds roughly to the change from elastic to plastic strain.

The effect of stress on intensity of magnetization of iron crystals in three crystallographic directions for different magnetic fields, was measured by Webster.† His results are shown in Fig. 83. They

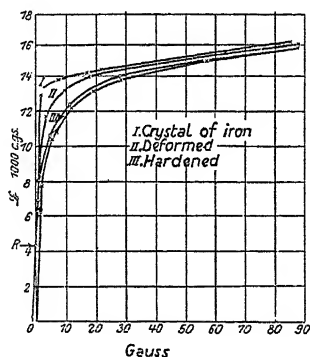


FIG. 82.

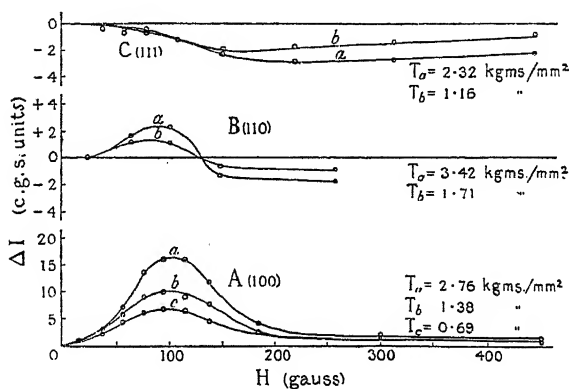


FIG. 83.

agree with the effect found for magneto-striction. When the direction of the axis of the specimen coincides with a $[100]$ crystal direction there is an increase in magnetic intensity corresponding to an increase in length with magnetization. For a $[111]$ direction the reverse is the case, and for a $[110]$ there is first an increase followed by a decrease.

In the papers of Becker and Kersten‡ a large number of experi-

† W. L. Webster, *Proc. Roy. Soc. A*, **109** (1925), 570.

‡ R. Becker, *Z. Physik*, **62** (1930), 253; R. Becker, M. Kersten, *ibid.* **64** (1930), 660; M. Kersten, *ibid.* **71** (1931), 553; **76** (1932), 505.

ments were carried out to test a theory of Becker regarding the relationship between mechanical and magnetic effects. In this theory he seeks an explanation of the magnetization process in the inherent stresses caused by elastic distortions.

Four nickel wires were tested.† *A* was a hard-drawn wire. *B* was annealed *in vacuo* at 800° C. *C* was annealed and extended in tension from 1.15 to 0.98 mm. cross-section. *D* was annealed and drawn

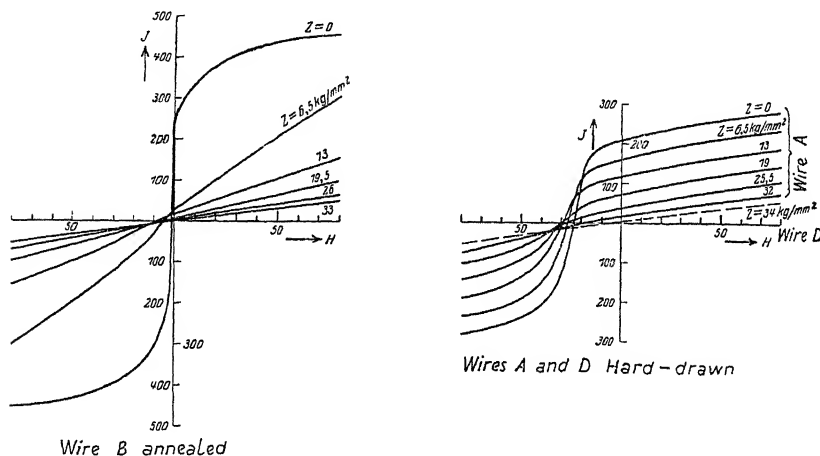


FIG. 84. Magnetization curves of nickel wires, hard-drawn and annealed.

through dies from 1.15 to 0.97 mm. cross-section. Each wire was then magnetized without and with increasing superimposed stresses. The maximum differences were shown by wires *A* and *B*, Fig. 84. The final effect of loading on all the wires seemed to be nearly the same. The intensity of magnetization became less together with the hysteresis, but the coercive force tended to become greater. The degree of distortion produced in the wires by wire-drawing or extension appears to have less effect on the magnetization curves than that produced by the load imposed during the experiment, but there are clearly two types of stress affecting the results. There are the initial, inherent stresses in the material and the stresses superimposed by loading. It is possible, according to these workers, to estimate the degree of initial internal strain from the permeability and susceptibility of the material. They estimate the internal stress of annealed nickel to be of the order of 0.8 kg./mm.² which corresponds to an

† R. Becker, M. Kerston, *Z. Physik*, **64** (1930), 660.

elastic extension of 4×10^{-5} . The magneto-striction of nickel was found to be 3.6×10^{-5} so that the strains are of the same order, in both cases. By measuring the permeability of wires subjected to different amounts of stress, they estimated the ratio of the initial stress to that imposed by tension and above the yield-point. The two are approximately proportional, σ_i being about $\frac{1}{4}\sigma$ at a stress of 35 kg./mm.^2 If this is correct, it means that the internal stresses of the material in the plastic range are about $\frac{1}{4}$ of the maximum possible obtainable stress. Fig. 85 shows the change in shape of the magnetiza-

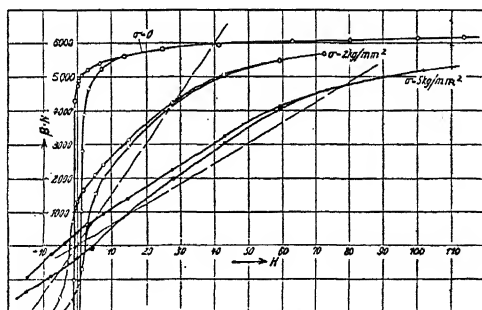


FIG. 85. Magnetization curves of nickel wires under different loads.

tion curve under different loads. It was shown that after the load had been removed and the material retested the coercive force had increased very nearly in proportion to the previously applied load.

Non-ferrous. Recent measurements on the effects of cold working on a number of non-ferrous metals has shown that the magnetic susceptibility of diamagnetic metals such as copper, silver, gold, and lead decreases by cold working. Copper actually changes from being diamagnetic to becoming paramagnetic. Fig. 86, which is due to Honda and Shimizu,[†] compares this change with the change in density due to the cold working, which in this case was produced by compression. On annealing, the reverse change takes place when crystallization begins. In connexion with these experiments, the authors also measured the susceptibility parallel and perpendicular to the direction of compression and found them to be the same.

The paramagnetic substances investigated, aluminium, platinum, rhodium, palladium, molybdenum, and barium, all showed a diminu-

[†] Honda, Shimizu, *Nature*, **126** (1930), 990; *Sci. Rep. Tohoku Imp. Univ.* **20** (1931), 460.

tion in susceptibility which was reversed on heating, with the exception of rhodium. Gerlach[†] has suggested that this phenomenon may be connected with the formation of amorphous material, but as Honda and Shimizu point out, there is no evidence for supposing that amorphous metal is paramagnetic and in any case it cannot be

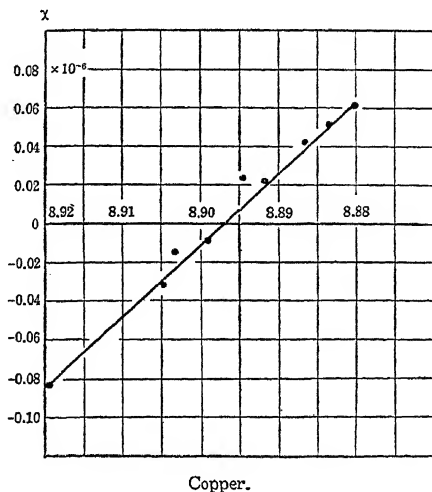


FIG. 86. Change in susceptibility compared with density changes due to cold work.

both more paramagnetic and more diamagnetic according to the metal from which it has been formed.

It may be mentioned that Banta's results[‡] do not agree with those given above, but the evidence is more in favour of there being such a change than against it.

It now seems probable^{§||} that the change in susceptibility of copper is due to the separation of a dissolved ferro-magnetic substance, e.g. iron, on working. If due allowance is made for the iron separation the paramagnetic susceptibility decreases and the diamagnetic susceptibility increases with increasing degree of residual strain.

[†] W. Gerlach, *Nature*, **127** (1931), 556.

[‡] H. E. Banta, *Phys. Rev.* **37** (1931), 634. See also F. Bitter, *Phys. Rev.* **36** (1930), 978; F. E. Lowance, F. W. Constant, *Phys. Rev.* **38** (1931), 1547.

[§] Y. Shimizu, *Sci. Rep. Tohoku Imp. Univ.* (1) **22** (1933), 915.

^{||} A. Kussmann, H. J. Seemann, *Z. Physik*, **77** (1932), 567.

*Other references*L. McKeehan, *Phys. Rev.* **36** (1930), 948.P. McCorkle, *Phys. Rev.* **25** (1925), 541.Smith, *Phys. Rev.* **4** (1914), 267.von Awers, *Wiss. Ver. Siemens-Konzern*, **9** (1930), 262.**5. Thermoelectric force between worked and annealed metal**

If a thermocouple is constructed from two wires of the same metal, one in the hard-drawn state and the other annealed, the potential difference between them can be measured. A number of metals in both rolled and drawn states have been investigated in this manner, chiefly by Borelius† and Thiele‡. The following table shows that sometimes the worked metal is positive with regard to the annealed, and sometimes negative. It also varies in the case of sheet, according to the direction of measurement in relation to the direction of rolling, the maximum difference being found in cadmium.

Thermoelectric Force per °C. in 10^{-8} volt, hard against soft

Metal	to direction of rolling	⊥ to direction of rolling	Drawn wires
Aluminium . . .	+28 +14	+40 +14	+42 +12
Lead	- 3	- 3	- 3
Gold	+ 8	+ 5	+ 7
Copper	+14 + 9	+12 + 8	+ 8 + 6
Platinum	-30 + 6	-24 + 6	-25
Silver	+58	+48	+55
Iron	-37	-35	-34
Nickel	+38	+23	+38
Cadmium	+ 4	+113	+17
Zinc	-25	+28	+24
Tin	+17	-11	± 0
Constantan . . .	-72	+32	-70

Thiele also observed that magnetizing iron, nickel, and cobalt affected the thermoelectric force and that the time factor was also important. Normally the current flows from the worked to the annealed metal, and this is also the case if both are magnetized. But if an annealed magnetized wire is compared with an annealed non-magnetized wire the current is reversed.

† G. Borelius, *Ann. Physik*, **60** (1919), 381.‡ J. Thiele, *ibid.* **72** (1923), 549.

6. Thermal conductivity and coefficient of thermal expansion

The changes in the thermal conductivity of aluminium wires produced by torsion have been studied by Calthrop.[†] Wires in the annealed and hard-drawn states and in the form of one crystal all showed a slight decrease in conductivity. The amount of torsion applied was not sufficient to produce a permanent twist. The thermal conductivity of all the wires was measured and found to be the same.

According to Bridgman,[‡] the effect of tension on the thermal conductivity of a number of metals, with the exception of nickel, is to cause a decrease in the conductivity. Nickel also proved an exception with regard to change in electrical conductivity. The loads used in extending the specimen were usually not sufficient to cause plastic deformation.

The influence of working on the coefficient of thermal expansion has been studied by Jubitz[§] and Masing^{||} who find that measurements made parallel with and at right angles to the direction of rolling do not differ for metals belonging to the regular system, but that there is a considerable difference in non-cubic metals. It is greatest in zinc and cadmium, although magnesium, antimony, and bismuth show it to a less extent. This property is closely related to the internal structure produced by rolling. (See Chapter V.)

7. Difference in rate of solution between worked and annealed metal

Osmond and Werth^{††} compared the rate of solution of worked and annealed iron wires in dilute hydrochloric acid and found that in 165 minutes 0.321 gm. of the worked metal dissolved compared with only 0.006 gm. of the annealed. Similar results were obtained with other acids.

Heyn and Bauer^{‡‡} also determined the solubility of drawn steel wires in sulphuric acid and of different parts of a low carbon steel test-piece in which the solubility was compared with the elongation.

[†] J. E. Calthrop, *Proc. Phys. Soc.* **38** (1926), 207.

[‡] P. W. Bridgman, *Proc. Amer. Acad. Arts Sci.* **59** (1923-4), 119. See also Smith, *Phys. Rev.* **28** (1909), 107; C. H. Lees, J. E. Calthrop, *Proc. Phys. Soc.* **35** (1923), 225; **36** (1924), 168.

[§] W. Jubitz, *Z. techn. Physik*, **11** (1926), 522.

^{||} G. Masing, *Z. Metallkunde*, **20** (1928), 425.

^{††} F. Osmond, Werth, *Ann. des Mines*, **8** (1885), 5.

^{‡‡} E. Heyn, D. Bauer, *J. Iron Steel Inst.* **79** (1909), 109. See also *Mitt. K. Materialprüfungsamt zu Gross-Lichterfelde* (1909), 57.

According to this investigation, the rate of solution first increases slowly with the elongation, then more rapidly, and then only very slowly as the stretching continues.

Goerens† compared wires in both states with similar results.

The difference in solubility and corrodibility between worked and annealed metal is very important in corrosion problems. It may be mentioned here that a cut surface‡ will corrode much more rapidly than one that has not been damaged, and a number of phenomena which are due to what may be termed 'corrosion fatigue' have recently been investigated.

Rosenhain and Murphy§ found that a mild steel boiler-plate bent backwards and forwards in various solutions tended to develop cracks more rapidly than if the same process was carried out in air.

A recent experiment|| with an aluminium crystal subjected to alternating torsion stresses while immersed in tap-water, showed that corrosion and the cracks that caused failure were definitely related to the stress distribution and slip in the crystal, although the pits which were also formed were not so related.

It therefore appears that internal stresses tend to accelerate solution of the metal.

8. Solution Potential

If two pieces of a metal, one in the worked condition and one annealed, are placed in a solution of one of its salts to form a cell, the hard metal is found to be electro-negative towards the soft and will tend to dissolve first. Spring†† measured the electromotive force between two samples of a number of metals in the two states. Tammann‡‡ has pointed out that, as the results depend entirely on the conditions of the surfaces of the specimens, the differences measured are not indicative of the degree of working of the whole volume of the specimen.

Goerens§§ measured the potential difference between a cold worked and an annealed specimen of the same metal (iron and steel) in a slightly acidulated decinormal solution of iron sulphate. The worked

† P. Goerens, *J. Iron Steel Inst. Carnegie Schol. Mem.* (1911), 320.

‡ U. R. Evans, *J. Soc. Chem. Industry* (1929), 97.

§ W. Rosenhain, A. J. Murphy, *J. Iron Steel Inst.* **123** (1931), 259.

|| H. J. Gough, D. G. Sopwith, *Proc. Roy. Soc. A*, **135** (1932), 302.

†† W. Spring, *Bull. Acad. Roy. Belg.* **41** (1903), 1066.

‡‡ G. Tammann, *Lehrbuch der Metallographie*.

§§ P. Goerens, *J. Iron Steel Inst. Carnegie Schol. Mem.* (1911), 320.

metal was negative relative to the annealed. Both further working or annealing altered the solution potential.

9. Change of colour of certain gold-copper-silver alloys on working, etc.

Tammann† has observed that alloys of silver and gold, containing 60–75 per cent. gold, which are white in the annealed state, become yellower and finally a deep yellow-green on rolling.

Another phenomenon is the reaction of copper-gold alloys to a sodium sulphide solution. In the annealed state there is a very sharp limit to the composition which is unattacked by the sulphide. In the worked state, the limit is less well defined and is displaced towards the gold side of the equilibrium diagram.

† G. Tammann, *Lehrbuch der Metallographie*.

XIII

PLASTICITY

ONE may distinguish two types of solid bodies which will yield and deform permanently in the solid state.

1. Amorphous substances such as pitch, certain waxes, glass, etc. are closely related to fluids. They have no fixed melting-point and their properties change in a uniform manner as the temperature is raised. A change of shape can be produced by quite small forces, often the weight of the body itself, given sufficient time. Like liquids, the particles of which they are composed are arranged in a random manner and are free to move in any direction equally according to the forces acting upon them.

2. A crystalline body differs from an amorphous body in that, although there is a gradual change in properties with temperature, there is a definite melting-point at which its crystalline properties disappear. The most important, from the present point of view, is cohesion and what we always associate with a crystalline solid, i.e. directional properties related to internal structure.

Most solids exhibit a certain degree of elasticity although it may be extremely small. Mathematicians and engineers have long studied the distribution of stresses and strains within the elastic range. The change from purely elastic distortion to purely plastic never takes place in metals. There is never an abrupt change from the one to the other—except possibly in the case of mild steel—and the plastic range is always complicated by work-hardening. These facts probably account for the reason why the conditions governing plastic deformation have not been investigated from the point of view of stress distribution in the same way as has been done so fully for the elastic range. It is only comparatively recently that the same methods have been applied to plastic deformation. Metals can be compressed elastically and will withstand high hydrostatic pressures† but sustain no permanent effects (with the exception of porous substances). On the other hand, a material can only withstand a simple tension, for example, up to a certain limit without failure. The problem is to find what is the state of stress in a body when it begins to yield plastically or to fracture. A number of theories have been put

† P. W. Bridgman, *Proc. Amer. Acad. Arts Sci.* 58 (1923), 166.

forward in order to account for the method in which a material will fail. These cannot be described fully here, but the following are the most important.

1. Maximum stress theory

According to this theory, the maximum principal stress in the material determines failure regardless of what the other principal stresses may be. This cannot be a true picture of the facts because (a) if this theory were correct, metals would yield under high hydrostatic pressure, (b) fracture of a test specimen would be expected to occur at right angles to the maximum stress.

2. Maximum strain theory (St. Venant's theory)

According to this theory the maximum positive elastic extension of the material in a stressed body determines failure by fracture or by plastic flow. This also is not borne out by experiment.

3. Maximum shear theory

Following some results on the extension of metals, Tresca, among others, concluded that failure should occur on those planes in the material subjected to the greatest shear stresses. This is often found to be the case in practice, the most striking example being the formation of Lüders lines in iron and mild steel. These have recently been studied by a large number of workers. On the other hand, brittle substances which do not yield plastically often fracture differently in tension and compression, and the observed values of their strengths are different.

Later modifications of the maximum shear stress theory are as follows:

1. *Mohr's theory*[†] or *Guest's law*.[‡] This states briefly, that the maximum shear stress determines the beginning of plastic flow independently of the other components of shear stress, i.e. when either T_1 or T_2 or T_3 rises to a certain value K . At the same time, according to Mohr, the shearing stress S_s in the planes of slip reaches, at the limit, a maximum value dependent on the normal stress S_n , acting in the same planes, and on the properties of the material. In this way he accounts for differences in yield-point in tension and compression and for the fact that the observed flow lines are not always inclined

[†] O. Mohr, *Z. Verein. deuts. Ing.* **44** (1900), 1524.

[‡] Guest, *Phil. Mag.* (1900), 69.

at the same angle in all substances. For example, in brittle substances there is a greater deviation from the calculated value of 45° .

2. *von Mises's theory.*† According to this theory, the sum of the squares of the principal shear stresses must rise to a given value before failure begins, i.e. plastic strain begins when $T_1^2 + T_2^2 + T_3^2$ rises to the value $2K^2$, K having the same meaning as in Mohr's hypothesis and being defined as half the lowest direct stress at which a tensile specimen can be extended plastically.

These theories will be found more fully described in the original papers or in the following: A. Nadai, *Plasticity*, McGraw-Hill Book Co., New York, and in numerous papers; S. Timoschenko and J. M. Lessels, *Applied Elasticity*, Constable & Co., London; *Proc. 1st International Congress for Applied Mechanics*, Delft, 1924.

These conditions are extremely difficult to determine experimentally. The work of Lode,‡ which has since been repeated and amplified by Taylor and Quinney,§ is the only attempt to subject a metal to varying ratios of principal shear stresses. Lode used metal tubes and subjected them simultaneously to direct tension and internal pressure. From the change in dimensions of his tubes, he was able to calculate the ratios of the principal shear stresses and compare them with the principal shear strains. In the diagram, Fig. 87,

$$\mu = 2 \left(\frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \right) - 1,$$

and

$$\nu = 2 \left(\frac{e_2 - e_3}{e_1 - e_3} \right) - 1,$$

where $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses so that $\sigma_1 > \sigma_2 > \sigma_3$, and e_1 is the extension of the material in the direction of the length of the tube, e_3 is that in the direction of the radius, and e_2 is the tangential extension obtained by measuring the change of mean radius of the tube. In pure tension $\mu = -1$, in pure shear $\mu = 0$, and in compression $\mu = +1$. It can be proved that symmetry alone necessitates that $\mu = \nu$ at these three points, as is shown in the diagram, Fig. 87. Lode's results, although probably only approximate, do indicate that μ does not equal ν over the whole range investigated. von Mises proposed that this relationship should hold as it does in the case of

† R. von Mises, *Nachr. Gesellsch. Wissensch. Göttingen* (1913), 582; see also H. Hencky, *Z. ang. Math. Mech.* 4 (1924), 323. R. von Mises, *Z. ang. Math. Mech.* 8 (1928), 161.

‡ W. Lode, *Z. Physik*, 36 (1926), 913.

§ G. I. Taylor, H. Quinney, *Phil. Trans. Roy. Soc. A*, 230 (1931), 323.

viscous fluids, but both Lode's, and Taylor's and Quinney's results are contrary to this hypothesis. In the latter investigation, thin-walled tubes were again used. These were subjected to simultaneous direct loading and to torsion, and the change in internal volume

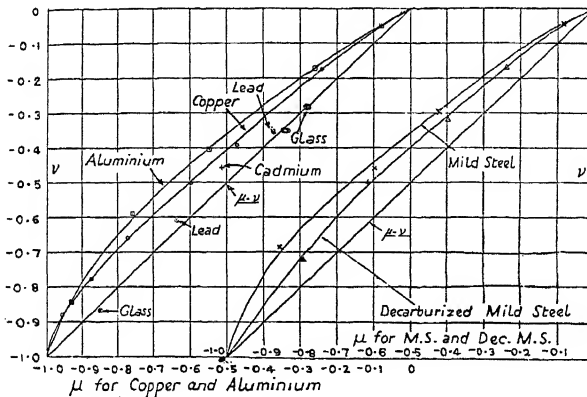


FIG. 87.

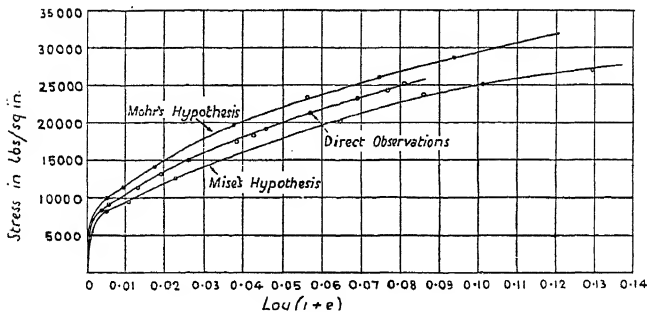


FIG. 88.

measured. Fig. 87 represents a summary of their results. In the case of glass, $\mu = \nu$ throughout the range, and the low melting-point metal lead lies very near the straight line $\mu = \nu$. All the other metals deviate from this line.

Any non-uniformity of the tubes, such as large crystal size, gave inconsistent results, with a still greater deviation from the $\mu = \nu$ relationship.

In another section, reference was made to the slightly different results obtained when metals were tested in tension, and compression

and torsion. Taylor and Quinney compared the curve obtained from direct loading with the torque-twist curves calculated according to Mohr's and von Mises's hypothesis respectively. Fig. 88 shows that these are different and that the method of derivation of the curve may well account for any differences observed in comparing tests carried out in both manners. It is assumed in both von Mises's and Mohr's theories that the resistance to further distortion depends on the amount of work done, which would be a function of K . The curves prove that this is not true in either case, although the differences are slight.

Another attempt to solve this problem was made by von Karman† on marble. He loaded cylindrical test-pieces in compression and subjected them at the same time to increasing hydraulic pressure. The curves so obtained vary from the type usually associated with a brittle substance, such as marble is under ordinary test, to curves representative of ductile metals. The internal structure confirmed these differences, the specimens deformed under high lateral pressures showing slip-bands and twin formation accompanying a general distortion of the grains like metals such as copper. In the normal compression test, there were fine cracks and a tendency for the grains to separate at the boundaries.‡

† von Karman, *Z. Verein. deuts. Ing.* 55 (1911), 1749.

‡ See also Johnston and Adams, *Amer. Journ. Sci.* 35 (1916), 205.

XIV

THE EFFECT OF HEAT ON DEFORMED METALS

No account of the plastic distortion of metal crystals can be considered to be complete without some reference to the changes in the structure brought about by heating deformed metal. Heating at a sufficiently high temperature for a suitable time removes most of the effects of deformation on the mechanical and physical properties of metals, and the distorted crystals undergo a process of recrystallization. The structure of a metal may be permanently altered by work and can only be obtained in the original condition if the metal is remelted. The heat-treatment of cold-worked metal plays a very important part in many industrial operations, such as rolling, drawing, etc., consequently the literature describing the effects of heat on deformed polycrystalline metal is very extensive, and it is not proposed to give a complete account of the subject. The main facts only can be summarized.

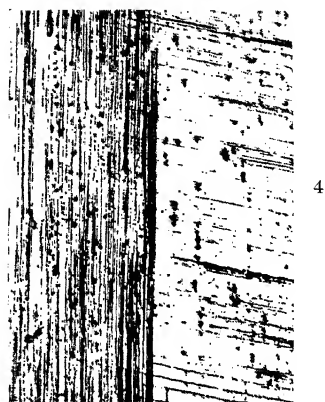
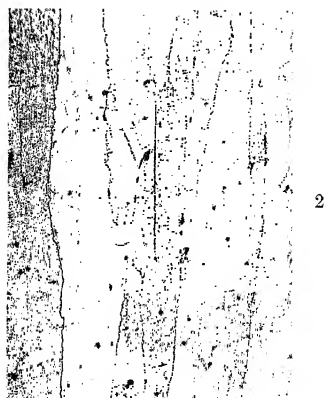
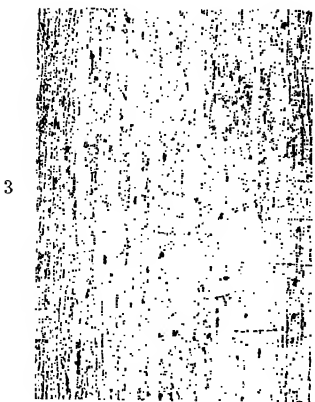
If a metal that has been plastically deformed is heated to a suitable temperature for sufficient time, the distorted grains are replaced by new crystals which form first in the boundaries of the old grains and gradually absorb them. The process is illustrated in Fig. 5, Plate V. The new crystals are no longer distorted in shape and are usually equiaxed. When recrystallization is complete, the crystals continue to grow at the expense of each other, until they have reached a size which is characteristic of the annealing temperature and of the amount of deformation. The gradual absorption of one crystal by its neighbours is illustrated in Fig. 6, Plate V. In this particular alloy each stage is registered by a permanent mark on the polished surface corresponding to the removal of the specimen from the furnace.

Existing evidence is against any recrystallization or crystal growth unless the crystals have been plastically deformed or undergo a polymorphic or phase change. The latter cause may also be considered as one connected with internal strain. Ewing and Rosenhain† first remarked upon the stability of metal crystals in an aggregate which has separated from the molten state, and subsequent attempts‡ to induce crystal growth in cast metals have resulted in failure. In

† A. Ewing, W. Rosenhain, *Phil. Trans. A*, 193 (1903), 353.

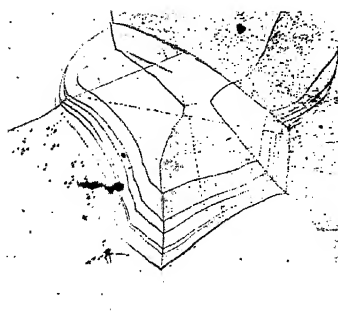
‡ H. C. H. Carpenter, C. F. Elam, *J. Inst. Metals*, 24 (1920), 83.

PLATE V



FIGS. 1, 2, and 3. Cast and rolled aluminium

FIG. 4. Slip-bands on aluminium crystal tested in alternating torsion



support of these negative results there is a great deal of experimental evidence connecting strain with crystal growth and recrystallization. If a number of test-pieces of soft annealed metal are taken and subjected to different amounts of extension or compression and subsequently heated to different temperatures the following facts are established:

1. There is no change until the metal has been plastically deformed.
2. There is no change until a definite temperature is reached.

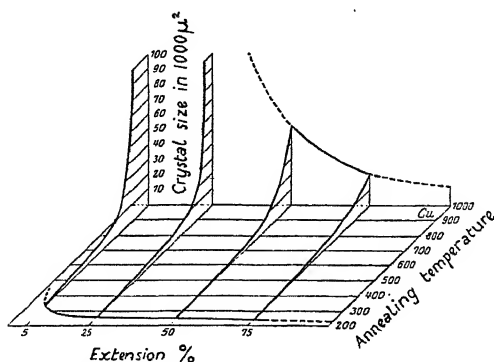


FIG. 89. Recrystallization diagram, electrolytic copper.

3. A certain minimum time is necessary for any change to take place.

4. The temperature at which any change occurs depends on the amount of deformation, being lower for large deformations and increasingly higher as the deformation diminishes.

5. The crystal size depends on the amount of deformation and the temperature of annealing, given sufficient time for the changes to be completed. The smallest amount of strain and the lowest annealing temperature, which will cause recrystallization at all, give the largest crystals. For large amounts of strain the highest annealing temperatures usually give the largest crystals, but the rate at which the metal is heated to the required temperature has some influence on the results. The relationships are illustrated diagrammatically in Fig. 89.

These facts are all in accordance with the formation on heating a deformed metal of strain-free crystal nuclei which subsequently grow at the expense of the distorted crystals because a strain-free crystal is more stable than one which is distorted. The smaller the deformation and the lower the annealing temperature, the smaller the number

of new nuclei formed and the larger the resulting crystal size. By suitably regulating the conditions, a test-piece consisting of many million crystals is entirely converted into one crystal.

After severe cold work another factor affects the crystal size. The new crystal nuclei have been described as strain-free, but that is probably not strictly true. If a specimen which has partly recrystallized is examined after suitable etching, the new crystals often appear to be less attacked by the reagent than the rest, so that relatively speaking, they appear freer from strain than the latter. This often provides a means of distinguishing them under the microscope. Another characteristic is their situation in the boundaries of the old grains or along twin and cleavage planes. They are therefore formed in regions of maximum stress. At high temperatures the atomic mobility is probably such, that the new crystals form and grow in comparative freedom. On the other hand, there is considerable evidence that, particularly at low temperatures, the effects of deformation persist during and even after the process of recrystallization. X-ray photographs† of severely worked metal which has been annealed at a low temperature, Fig. 6, Plate IV, show that the preferred orientation due to working persists in the new crystals, although to a less extent. At high temperatures crystals of random orientation are produced.‡ Somewhat similar results are obtained with single crystals.§

Again, the shape of the crystals is sometimes influenced by the direction of previous work; for example, they may be elongated in the direction of rolling.||

After recrystallization is complete, that is to say, when none of the original crystals remain, growth of the new crystals at the expense of each other still goes on at high temperatures and the final crystal size also depends on the amount of original deformation.†† If strain is the cause of recrystallization and crystal growth—and there does not appear to be any explanation that fits the facts so well—then it follows that the new crystals themselves are not entirely free from strain, and this is not surprising if one considers that a rearrangement

† R. Glocker, *Z. Physik*, **14** (1923), 328; **16** (1923), 314. Glocker, Kaup, Widmann, *Z. Metallkunde*, **17** (1925), 354.

‡ Cf. Chapters V and IX.

§ W. G. Burgers, *Z. Physik*, **67** (1930), 605.

|| H. C. H. Carpenter, C. F. Elam, *J. Inst. Metals*, **25** (1921), 259.

†† W. Feitknecht, *J. Inst. Metals*, **35** (1926), 131.

in the solid state is probably more difficult to accomplish than one in the molten. But the phenomenon of *growth* following strain is somewhat different from recrystallization due to the formation of new crystals. There is some evidence† that a crystal itself in an aggregate may be induced to grow if the specimen is strained a lesser amount than would be necessary to bring about recrystallization by the deposition of new nuclei. This may occur in one of two ways. Either one crystal remains less strained than its neighbours and grows at their expense; or the most strained crystal grows, either directly or by first recrystallizing itself. It has been shown‡ that an aluminium crystal of a certain orientation can be converted into another of a different orientation by suitable straining and heat treating. The effects of the deformation shown in the mechanical strength and character of the X-ray reflections are lost in the process. If the degree of distortion exceeds a certain value the crystal breaks up into several large crystals of different orientation, and as the distortion increases, the crystal size diminishes.§ Hence, any or all of these processes may take place in an aggregate, but it is extremely difficult to decide which is the most probable, because the results are so similar. These problems have been discussed in a number of papers, but no entirely satisfactory conclusion has yet been arrived at.

Other factors which have been thought to promote recrystallization and crystal growth are difference in crystal size and orientation, and irregularity of boundary. All these seem to be of minor importance compared with the effect of mechanical deformation.|| A possible mechanism of crystal growth is referred to in discussing the other changes brought about by plastic deformation (Chap. XV).

It does not seem necessary here to give a detailed account of the effects on the structure as revealed by X-rays during the process of recrystallization. The first effect is that the resolution of the K_{α} doublet is regained at temperatures below those necessary to bring about mechanical softening and recrystallization (p. 127). The elongation of the Laue spots, however, remains until recrystallization takes place.

† H. C. H. Carpenter, C. F. Elam, *J. Inst. Metals*, **24** (1920), 83. W. Feitknecht, loc. cit.

‡ H. C. H. Carpenter, C. F. Elam, *Proc. Roy. Soc. A*, **107** (1925), 171.

§ W. G. Burgers, J. C. M. Basart, *Z. Physik*, **51** (1928), 545; **54** (1929), 74.

|| A general summary of these problems may be found in the papers mentioned above, together with many references and accounts of much experimental work.

The next step is the appearance of spots in the diffuse Debye-Scherrer rings of the typical cold-worked structure and, as has been noted already,[†] the spots are concentrated at low temperatures in regions of maximum intensity of the original pattern. Therefore certain directions in the first crystals to be formed, approximate in orientation to those of the worked metal. The final orientation of the recrystallization structure is not the same although related to it. A preferred orientation sometimes disappears entirely after high-temperature annealings, but in copper it persists up to the melting-point.[‡]

Burgers§ has carried out a very extensive investigation into the number and relative orientation of the crystals produced when large aluminium crystals are compressed. He finds that they can be related to the original orientation and to the deformation, particularly if it is assumed that a rotation of crystal blocks, such as Taylor and Yamaguchi|| have suggested, takes place. The agreement is only approximate. The number of crystals formed depends on the amount of shear and are fewer in number when two slip-planes function than when the whole of the distortion takes place on one plane.

A very important factor in the results of physical and mechanical tests is the period for which the deformed metal has been heated before testing. At high temperatures the maximum effect may be obtained in a few minutes. The lower the temperature the slower the changes take place. For example, it has been shown^{††} that cold-rolled aluminium heated at 500° C. loses nearly all its work-hardness in the first hour; at 250° C. it takes 800 hours; while at 200° C. loss was not complete at the end of 49,500 hours (5.65 years). A pure metal softens more rapidly than an impure one, and also usually recrystallizes at a lower temperature for the same degree of deformation.

Observations of the time taken for large crystals to grow in the solid state after deformation are recorded by a number of workers.^{‡‡} This again depends on the temperature, the degree of previous strain, and the purity of the metal. A quantitative estimation§§ of the rela-

[†] R. Glocker, loc. cit.; cf. Chapter V.

[‡] Von Göler, G. Sachs, *Z. Metallkunde*, **19** (1927), 90. E. Schmid, G. Wassermann, *Z. Physik*, **40** (1926), 451.

[§] W. G. Burgers, *Z. Physik*, **59** (1930), 651; **67** (1930), 605; **81** (1933), 43.

^{||} Cf. Chapter X. ^{††} H. C. H. Carpenter, C. C. Smith, *J. Inst. Metals*, **29** (1923), 29.

^{‡‡} H. C. H. Carpenter, C. F. Elam, loc. cit. W. Feitknecht, loc. cit.

^{§§} R. Karnop, G. Sachs, *Z. Physik*, **60** (1930), 464.

tion existing between the different factors has been attempted, but these do not appear to be of a simple character.

The importance of the part played by time in tests carried out at higher temperatures and over long periods has already been stressed (Chap. IX).

Change in mechanical properties

The first effect of heat on the mechanical properties of worked metals is frequently to raise the elastic limit, yield-point, and breaking-

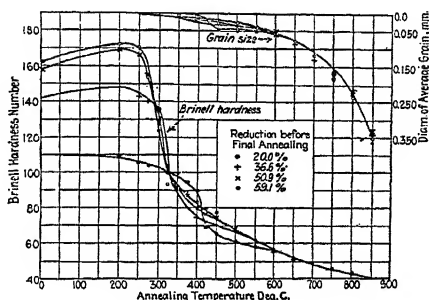


FIG. 90. Effect of heating on Brinell hardness of cold-worked brass. (Bassell and Davis.)

stress.† The influence of low temperature annealing in raising the elastic limit of overstrained steel has already been noted (p. 90). It is attributed to the release and redistribution of internal stresses caused by inhomogeneity of straining.

The metal then begins to soften at a rate depending on the temperature and degree of deformation. Some mechanical softening certainly takes place before the birth of new crystals. It is frequently associated with a somewhat granular appearance if the specimen is suitably etched.‡ During the process of recrystallization softening sets in very rapidly§ and most of the effects of work disappear when the metal has recrystallized completely (Fig. 90). In most cases rapid softening after cold-work takes place at temperatures closely approximating to those at which the metal loses mechanical strength.

† Charpy, *Contribution à l'étude des alliages*, 1901, pp. 1-31. G. D. Bengough, O. Hudson, *J. Inst. Metals*, 4 (1910), 92. C. H. Mathewson, A. Phillips, *Bull. Amer. Inst. Min. Eng.* (1916), 1-50.

‡ H. C. H. Carpenter, C. F. Elam, *J. Inst. Metals*, 25 (1921), 259.

§ Cf. Fig. 70, Chapter IX.

Changes in physical properties which accompany structural and mechanical changes are described in Chapter XII.

There are certain effects of deformation and heating which are permanent. A coarsely crystalline or dendritic structure is broken up and, in the case of alloys, homogeneity is obtained much more rapidly than by simple annealing. If the deformation is carried out at temperatures above which a second constituent becomes soluble, subsequent annealing leads to its separation in an entirely different form and it is usually more evenly distributed. These are matters of great practical importance, but are outside the scope of this book.

THEORETICAL DISCUSSION AND SUMMARY

It is not surprising that many theories have been put forward to explain the changes that occur in the properties of metals, when they are plastically deformed. Before describing some of these and showing how far they go towards a proper understanding of the problem, it is desirable to summarize what it is required to explain. In the first place it must be recalled that hydrostatic pressure, i.e. pressure in all directions, does not permanently alter the properties, except by closing up pores, cracks, etc. Plastic deformation, however, produces the following changes:

1. A change in elastic properties, which involves raising the elastic limit and the acquiring by the metal of a form of greater elasticity. It is doubtful whether a metal ever obeys Hooke's law, since greater refinements of measurement indicate greater deviation from it. A repeated loading and unloading in the region of plastic deformation produces a 'cyclic' state (see p. 93) in which the metal again becomes elastic, but the loading and unloading curves enclose a loop, which for a given load becomes constant in area. The work done in each cycle is equivalent to the heat developed. Elastic creep is probably associated with this phenomenon.

2. Plastic deformation increases the resistance to further deformation.

3. The work done in producing a certain deformation is partly converted into heat, but a proportion of it, from 5 to 13 per cent., is absorbed by the metal in some form of stored energy.

4. A metal which has been plastically deformed will recrystallize if heated for a given time at a suitable temperature. By so doing, its properties will be restored to their original condition.

5. The physical properties are changed as follows:

- (1) The density decreases.
- (2) The electrical resistance increases.
- (3) The magnetic properties are altered. (a) Ferro-magnetic substances. The susceptibility, permeability, and remanent magnetism are lowered. The coercive force and hysteresis losses are increased. (b) Non-ferrous. Magnetic susceptibility of

diamagnetic substances decreases. Magnetic susceptibility of paramagnetic substances also decreases.

- (4) There is a thermoelectric force between worked and annealed metal.
- (5) The thermal conductivity is slightly reduced.
- (6) Worked metal is more easily dissolved than annealed and more easily corroded.
- (7) There is a difference in solution potential, the hard metal being electro-negative towards the soft.

6. Deformation of a crystalline material occurs

(a) by a process of gliding on a crystal plane or planes, determined by stress considerations, and in a definite crystallographic direction. Both slip-plane and direction of slip are determined by the type of lattice. The slip-plane is usually the plane of greatest atomic density and the direction of slip is parallel to the line of closest atomic packing in that plane. The number of planes functioning and the amount of slip on each of them are both variable quantities. The total number of slip-planes observed is small compared with the *possible* number.

The orientation of the crystal changes gradually relative to the direction of applied stress.

(b) By means of twinning. Here also gliding is on a definite plane and in a definite direction, but in order to obtain a twinned structure, every plane must move a definite amount. A sudden change of orientation is produced.

7. Plastic deformation is accompanied by changes in the character of X-ray reflections. These are of two kinds: (1) In most metals resolution of the K_{α} doublet is lost on straining. Annealing at temperatures lower than that required to produce mechanical softening, however, results in recovery. (2) Laue spots become elongated and the angle over which reflection of characteristic radiation occurs is increased. This indicates a bending of the crystal planes, although the degree of bending is not uniform throughout and the curvature may be greater in some directions than others. Not only do the reflections indicate a bending of the planes, but in some cases a rotation of fragments of the crystalline material through different angles about a common axis.

The space lattice of the material is unaltered in dimensions, except to an extent that can be accounted for wholly as elastic distortion.

A distortion of the space lattice is indicated by a broadening of the lines of the spectrum obtained with monochromatic X-rays. It has been found that certain lines show a greater degree of distortion than others.

8. The hardening produced by deformation is not confined to the actual slip-plane, but is equally distributed throughout the crystal. This is seen quite clearly when slip changes over from one plane to another. There is no apparent break in the stress-strain curve at this point and, in certain cases, slip will proceed on one plane after the shear stress on a second possible slip-plane is greater than on the first.

9. The addition of a second metal has many of the effects of work-hardening. The resistance to deformation is increased; the ductility is decreased (with some exceptions); the electrical resistance is increased; the solubility and corrodibility are usually increased; etc. Alloy crystals, however, usually give perfect X-ray reflections, showing no distortion until—like crystals of pure metals—they are deformed mechanically.

It is quite clear that in the present state of our knowledge no simple explanation is likely to be found to account for all the results of plastic deformation, although several of them are obviously inter-related, and none of them appear to be inconsistent.

The theories which have been put forward to account for these phenomena are of three types.

1. There is a definite change of state of the material, i.e. it ceases to be crystalline and becomes amorphous.

This theory was first put forward by Beilby† from a study of the structure of polished surfaces. He conceived the idea of a partial melting of the material on the planes of slip, due to the increased pressure.‡ In this state the parts glide over each other easily, but when the partially melted layer solidifies, it does so, not according to the original lattice, but in the form of an amorphous material resembling a glass. Such a material has certain specific properties. It is extremely hard and brittle under quickly applied stresses, while yielding gradually when the stress is applied slowly. Being in the nature of a viscous fluid, it is more elastic than a crystalline solid. It is less dense than the crystalline substance, and has a higher electrical resistance. On heating, it will recrystallize and consequently the properties of the metal will become normal. The theory thus provides

† Beilby's work is summarized in *Aggregation and Flow of Solids*, Macmillan & Co., 1921.

‡ See also Johnston and Adams, *Amer. J. of Sci.* 35 (1916), 212.

a satisfactory explanation of many of the peculiarities of worked metals. It is impossible to prove the existence of such layers by means of X-rays, but the fact that X-rays have detected a different kind of distortion in the slip-planes, is in itself contrary evidence.

Moreover, it has been shown that the direction of slip in a crystal plane is determined by lines of atoms in that plane, and this holds true up to very large amounts of distortion. The moment that the surface layer melted, these directions would presumably be obliterated, and the direction of slip would be the direction of greatest slope in the slip-plane. It would also be expected that the planes upon which slip is taking place, having a greater amount of amorphous material, would be harder than other similar crystal planes, but this is not the case. Plastic deformation hardens the metal as a whole.

It should be remembered that when Beilby's theory was promulgated, the evidence obtained from the work on metal crystals and from X-rays was not available. Moreover, the theory was very fruitful in stimulating research, and its extension to an explanation of the properties of crystal boundaries by Bengough and Rosenhain† has been of great value. In fact, both aspects of the theory are still made use of in explanation of some of the mechanical properties of metals, particularly at higher temperatures.

2. That change of orientation accounts for change in mechanical properties.

Crystals, being anisotropic solids, have different properties in different directions. This is very much marked in the crystals of lower symmetry.

According to Tammann‡ there are only two possible means of deformation, i.e. by slipping or by twinning. A crystal is plastic or not according to the number of glide-planes that can be formed. This depends partly on the orientation of the crystal towards the stress and on the structure of the crystal itself. A crystal which develops a large number of glide-planes will be more plastic than one which develops a few. His preliminary conditions for plasticity are formulated as follows: (1) the glide-plane system must be at least three; (2) the breaking strength of the crystal within the glide-planes must be greater than the force required to produce gliding. Slip will begin on those planes which show least resistance and those which

† Cf. Chapters V and IX.

‡ G. Tammann, *Lehrbuch der Metallographie*, Leipzig, 1923.

are more resistant will only function later. This, in effect, is also the view of Körber,[†] as an explanation of hardening by cold work and the formation of a fibre-structure.

The change of orientation which is produced in both single crystals and polycrystalline material has been described in Chapters II and V. Körber maintains that those planes along which gliding occurs are most easily rotated into less favourable positions. A greater force is therefore required to produce further deformation, and at the same time, by repetitions of this process, the crystals become definitely orientated relative to the stress. The part played by change of orientation as a contribution to the increased resistance to shear has been clearly demonstrated[‡] (Fig. 20, p. 36), and it is relatively small.

Experimental results are all in favour of an increased resistance to shear quite independent of that produced by the decrease in the angle between slip-plane and direction of applied stress. Unless this were so, failure would result immediately on the formation of the first slip-plane, because of the increase in tangential stress in the direction of slip caused by the decrease in angle between the axis of the specimen and the direction of slip.[§] The material would go on slipping at the point where it started and would break without extension having taken place in the rest of the test-piece. Such, in fact, must be the actual process in the failure of those crystals which break by cleavage along the slip-plane, although it has been shown that the stresses normal to the plane^{||} are most important in producing the final parting of the material.

It has been shown by Wood^{††} that the appearance of preferred orientation in rolled copper and Mumetal (70 per cent. Ni, 25 per cent. Fe, 5 per cent. Cu) does not take place until after a reduction in thickness of from 35 to 50 per cent. We know that very much less deformation than this produces hardening, so that here, in polycrystalline metal, the evidence is against the theory that work-hardening can be explained by means of orientation changes.

On the other hand, the differences in properties in different directions of rolled sheets, for example, zinc,^{‡‡} must be attributed to orientation of the crystals in the sheet relative to the direction of

[†] F. Körber, *Stahl und Eisen*, **48** (1928), 1433.

[‡] E. Schmid, *Z. Physik*, **22** (1924), 328.

[§] G. I. Taylor, C. F. Elam, *Proc. Roy. Soc. A*, **108** (1925), 28.

^{||} E. Schmid, *Proc. 1st Intern. Congr. Appl. Mech.* Delft, 1924.

^{††} W. A. Wood, *Phil. Mag.* **14** (1932), 656.

^{‡‡} Cf. p. 67.

rolling. These differences are small when the metals have high crystal symmetry.

3. The crystal structure is distorted and broken up in some form.

All the results of plastic deformation, physical, mechanical, and the effect on X-ray reflections, are consistent with some form of breaking up of the crystal structure, but the details of the distortion are not yet clear.

In the first place, one can assume that no crystal is perfect, even before deformation. Slight irregularities in solidification, the presence of impurities, and cracks due to contraction on cooling† all cause a disturbance in the regular arrangement of the atoms. A more regular form of discontinuity is that suggested by Zwicky.‡ His theory is based on the work of Lennard Jones.§ The atoms on the surface will try to arrange themselves in a lattice characterized by spacings smaller than the spacing of the solid crystal. Such a tendency will lead to cracks on the surface and block formation within the crystal. Some experimental confirmation of a mosaic structure in crystals has been obtained by Goetz|| in the case of bismuth. A perfect cleavage plane (111) showed fine parallel markings intersecting at 60°. For all the crystals examined, there was always a *smallest* triangular pattern that remained constant within the limits of error. This was also confirmed by etching.

Smekal's†† idea of defects in the crystal, which he considers must of necessity occur irrespective of the presence of impurities, has a similar application, particularly in an effort to explain the large discrepancy between the calculated strength of materials and that actually determined by experiment. For example, according to Traube,‡‡ the intrinsic pressures of metals vary from 3,340 kg./sq. mm. for nickel and 3,330 kg./sq. mm. for iron to 530 kg./sq. mm. for lead, whereas the breaking strength of hard steels is rarely more than 300 kg./sq. mm. Similarly, in the case of rock salt, the calculated value based on the lattice theory is 200 kg./sq. mm.§§ while the tensile strength determined experimentally is only 450 gm./sq. mm.||||

† P. Kapitza, *Proc. Roy. Soc. A*, **119** (1928), 358.

‡ F. Zwicky, *Proc. Nat. Acad. Sci.* **15** (1929), 253; **16**, 816.

§ J. E. Lennard Jones, B. M. Dent, *Trans. Faraday Soc.* **24** (1928), 92.

|| A. Goetz, *Proc. Nat. Acad. Sci.* **16** (1930), 99.

†† A. Smekal, *Phys. Zeitschrift*, **26** (1925), 707. *Ann. Physik*, **83** (1927), 1202. *Z. techn. Physik*, **203** (1927). See also R. Becker, *Phys. Zeitschrift*, **26** (1925), 919.

‡‡ Traube, *Z. anorg. Chem.* **34** (1903), 413.

§§ F. Zwicky, *Phys. Zeitschrift*, **24** (1923), 131.

|||| A. Joffé, etc., *Z. Physik*, **22** (1924), 286.

Griffith† has calculated the stresses that may arise at the end of cracks and fissures and finds them to be of a very high order, in fact, closely approaching the theoretical rupture strength of the material. He concludes that the presence of some such discontinuities accounts for the low experimental results obtained, and confirms this by preparing glass and silica rods by a special process to avoid surface cracks, etc., and finds that these have approximately the theoretical breaking-stress.

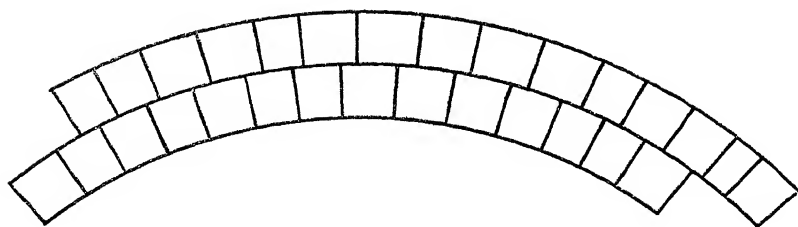


FIG. 91. 'Biegegleitung.'

The breaking up of the crystal shown by the evidence obtained from X-rays may have more than one interpretation.

The elongation of the Laue spots, first described by Czochralski,‡ was also interpreted by him as indicating a gradual distortion of the lattice involving alteration in the spacing of the planes ('Verlagerungstheorie'). Photographs of severely worked metals§ show that this is not true, the spacing of a particular substance remaining constant except for a small distortion within the elastic range. The only other possible explanation is that the planes are bent by breaking up into a system of blocks, which are nearly parallel, each block being itself undistorted. (Fig. 91.) This idea was applied to metals by Mark, Polanyi, and Schmid,|| although Mügge†† first used the term 'Biegegleitung' in connexion with minerals. According to Polanyi,‡‡ the bending is purely elastic, but the effect on the X-ray reflections is the same in either case.

Evidence of a still greater amount of breaking up is obtained from the fact that the reflected spots from a perfect crystal are replaced by

† A. A. Griffith, *Phil. Trans. Roy. Soc. A*, **221** (1920), 163. *Proc. 1st Intern. Congr. Appl. Mech.* Delft, 1924.

‡ Czochralski, *Z. Metallkunde*, **15** (1923), 60.

§ M. Ettisch, M. Polanyi, K. Weissenberg, *Z. Physik*, **7** (1921), 181.

|| H. Mark, M. Polanyi, E. Schmid, *Z. Physik*, **12** (1923), 58.

†† O. Mügge, *Neues Jahrb. Miner.* **1** (1898), 71.

‡‡ G. Masing, M. Polanyi, *Ergebn. exakt. Naturwiss.* **2** (1923), 177. Cf. p. 125.

Debye-Scherrer rings when the crystal is severely worked, and cutting and polishing† the surface have also the same effect.

A similar type of distortion is that described by Taylor and Yamaguchi‡ in which crystal fragments are actually rotated on the slip-planes. Fig. 92, which is due to Burgers,§ illustrates the kind of

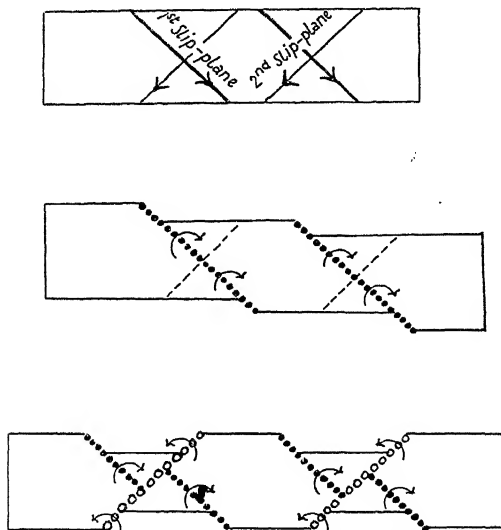


FIG. 92. Rotation of fragments on slip-plane for single and double slipping.

movement suggested. Yamaguchi has even shown that the amount of rotation increased with the amount of shear and was greater in hard crystals than soft.

It remains to see how such distortions could produce the necessary change in properties. As Joffé|| himself has pointed out, if the bending of the crystal planes is such that the direction of gliding coincides with the axis of bending there is no reason to suppose that the bent planes would not glide over each other as easily as the unbent. Bending of the slip-plane itself, therefore, need not cause a greater resistance to shear. There must be an elastic distortion in every lamellar slip such that the convex side is extended and the concave contracted, and the surface of separation must be in a state of instability because of the strains set up, but there is no clear picture of what may

† W. Boas, E. Schmid, *Naturwiss.* **20** (1932), 416.

§ W. G. Burgers, *Z. Physik*, **68** (1930), 605.

‡ See p. 126.

|| Joffé, *Physics of Crystals*.

be the cause of hardening on such a plane. The planes that cross these bent slip-planes are in a different category. There may be a definite hindrance to slip caused by their being out of alignment.

It is necessary to consider in what way irregularities in the slip-planes could hinder slip. The most simple explanation is that slip proceeds normally until two irregularities come together, when they interlock and prevent further movement on that plane. This theory was first suggested by Jeffries and Archer.[†] The presence of impurities, alloying atoms or crystal fragments of differing orientation, could all act as 'keys'. Crystal boundaries would function in the same way. Neglecting the possible effect of impurities, fragmentation could only account for part of the increase in strength, particularly in metals of cubic symmetry where a number of slip-planes and slip-directions are available. Taylor[‡] states that mathematical analysis shows that, however the fragment is orientated, the resistance to shear parallel to the plane of slip of the main body of the crystal cannot be greater than about twice the minimum value. Further, if the degree of non-alignment obtained from X-ray measurements is determined, the increase in the resistance to shear cannot amount to more than 2 to 3 per cent. of the original resistance of the unbroken crystal.

Locking caused by impurities or foreign atoms on the slip-plane would not involve the distortion shown by X-rays, nor does an explanation of hardening depending on mechanical causes such as this also explain the other characteristics of plastic deformation listed above.

These objections also apply to all the theories which depend on fragmentation only as an explanation of work-hardening. For example, hardening during fatigue tests is explained by Gough, Hanson, and Wright[§] as being due to 'buckling' or 'rumpling' of the slip-plane due to the formation of a number of crystal fragments, probably more perfect than the original crystal as a whole, but all of slightly differing orientations. The greater the deformation the more the breaking down of the structure, and at each stage the crystal fragments would be more 'perfect' and of greater stability under shear strain, than the surrounding material.|| This introduces a somewhat different idea.

[†] Z. Jeffries, R. S. Archer, *Science of Metals*.

[‡] G. I. Taylor, *Trans. Faraday Soc.* **24** (1928), 122.

[§] H. J. Gough, D. Hanson, H. Wright, *Phil. Trans. A*, **226** (1924).

|| H. J. Gough, *Trans. Faraday Soc.*, **24** (1928).

The less perfect crystal planes will tend to yield first. As the stress increases, more planes function, the material that has not slipped being still more perfect and having a tenacity more closely approaching the theoretical value of the material. Such a theory, however, does not explain why slip, having once begun on the less perfect planes, ceases. That fact is the essence of work-hardening. It may be recalled also, that whereas Yamaguchi found that the number of slip-bands produced on an aluminium crystal was proportional to the stress, it was not proportional to the amount of shear, particularly in the earlier stages of the test. In other words, although new slip-bands were continually being formed, some slip must have taken place on previously formed slip-bands.

An explanation of break-down due to fragmentation is more easily arrived at. As soon as the fragments become rotated so that their non-alignment exceeds a certain value, a crack will form which will spread due to excessive stress concentration at the ends, in the manner described by Griffith and others.

The other most important change in the X-ray reflections is the broadening of the lines in a Debye-Scherrer spectrum. This may be due to† (1) a gradual change in the lattice spacing throughout a single grain; (2) an exceptionally small grain size; and (3) distortion in certain regions of the crystal only. (1) may be due to variations in composition of solid solutions, for example; or to elastic distortions caused by loading below the yield-point; or to internal stresses, such as those causing 'season cracking', etc. With regard to (2), the influence of grain size on the width of the Debye-Scherrer lines does not make itself felt until the grain size is about 10^{-4} cm. It then becomes rapidly more marked as the structure becomes finer.‡ It is unlikely that the crystal grains are so small, except in very severely worked metal.

Accordingly, the most probable cause is (3), but here again the details of the distortion are obscure, although a number of suggestions have been put forward.

It is necessary at this stage to inquire what are the factors causing gliding to take place. There is a certain critical shear stress, whose value depends only on temperature, at which gliding begins and this has been shown to be independent of the stress normal to the slip-

† U. Dehlinger, *Z. Krist.* **65** (1927), 615; *Z. Metallkunde*, **23** (1931), 147.

‡ M. V. Laue, *Z. Krist.* **54** (1926), 115. See also C. G. Darwin, *Phil. Mag.* **43** (1922), 800.

plane. According to Frenkel,† work must be done in separating the planes before gliding takes place, but this is contrary to the evidence above, namely, that stresses normal to the slip-plane do not affect the critical shear stress. Becker‡ assumes that the combined stresses due to thermal motion and applied stresses reach a certain critical value before gliding begins. He imagines the structure to be divided into small elements of a definite volume, and gliding to begin in these regions. The size of these blocks is about 37 Å^3 and they contain about 3,000 atoms. They may be compared with the mosaic structure which Smekal also supposes to exist. An association of thermal stresses with stresses produced by externally applied forces is of value in helping to explain recrystallization phenomena and diffusion. There is nothing, however, in such a theory to account for the effects of plastic deformation. It is concerned more with conditions initiating gliding.

In any movement within the crystal resulting in permanent deformation, there must be a breaking of the bonds between layers of atoms and the forming of new bonds in order to maintain cohesion of the material. The work done in breaking the bonds is converted into heat, with the exception of a small percentage which it is supposed remains in the metal in some form of stored energy. The presence of quite large stresses and internal strain in worked metals has been proved to exist by Heyn and Bauer,§ amongst others, and are the cause of what is known as 'season cracking'. Closely associated with the presence of internal strain are the phenomena of elastic hysteresis and after-working. It has been shown by v. Wartenburg|| and by Joffé†† that a perfect crystal does not exhibit elastic hysteresis. One may therefore assume that it is associated with some kind of displacement of the atoms leading to a different stability within the lattice,‡‡ but not to loss of cohesion.

Both Prandtl§§ and Dehlinger||| have put forward somewhat similar theories on these lines. If a crystal is divided, the line of separation must pass between two lattice planes, and the atoms on the surface

† J. Frenkel, *Z. Physik*, **37** (1926), 572.

‡ R. Becker, *Phys. Zeitschrift*, **26** (1925), 919.

§ E. Heyn, O. Bauer, *Int. Z. Metallog.* **6** (1910), 16.

|| H. v. Wartenburg, *Verh. deuts. phys. Ges.* **20** (1918), 113.

†† A. Joffé, *Ann. Physik*, **20** (1906), 919.

‡‡ See also R. Becker, *Z. Physik*, **33** (1925), 185.

§§ L. Prandtl, *Z. ang. Math. Mech.* **8** (1928), 85

||| U. Dehlinger, *Ann. Physik* (1929), 749.

of each 'part' will be displaced, since they are no longer subject to the same forces as they were in the undisturbed lattice. Prandtl assumes that the atoms which lie between the regularly developed parts of the lattice are attached to one or other part according to the forces acting on them. They are supposed to be in a stable position at one particular place and are attracted back to this place when they are drawn away from it by other forces. The other half of the lattice exerts a force of a periodic nature resembling the forces which exist at the surface of a crystalline substance. Prandtl assumes the relation between these two forces to be such that there are at least two positions of stable equilibrium for an atom, given a suitable displacement for both parts of the lattice. This may be illustrated by Fig. 93. Here the force with which, by means of the absorbed atoms, the upper lattice acts on the lower, is plotted as a function of the displacements ξ . C and C' are fixed on the lower lattice. The curve consists of two stable portions and one unstable, the two stable paths corresponding to the movement of the atom in a hysteresis cycle. The change-over is determined by the relationship existing between the forces acting on the atom by both halves of the lattice.

In this theory elastic hysteresis and after-working are associated with irregularities in atomic arrangement, i.e. they do not occur in undeformed crystals, and this is in accordance with experiment.

Dehlinger's theory was first formulated to explain the facts of crystallization and crystal growth, but has much in common with Prandtl's and gives a possible picture of the distortion produced in a lattice by mechanical means.

At the surface of separation between two parts of a crystal or at a crystal boundary, the atoms are not in equilibrium. Should one of these atoms be displaced a little more, it will increase the gap between it and its neighbour and cause this atom to move. A periodic displacement of this type will travel across the surface of the lattice, causing a number of atoms to pass over into positions of greater stability, but at the same time causing an increase of instability in neighbouring rows of atoms.

The boundary between two parts of a crystal is illustrated in Fig. 94. The wavy line represents the potential force which the lower lattice exerts on an atom in the upper lattice. Here also there are two positions of equilibrium, according to the position of the atom on the potential curve.

Although these theories do not give a complete explanation of the effects of plastic deformation on the physical and mechanical properties they go some way towards it. Non-uniform displacements of atoms also entail the setting-up of internal strains. Heyn and Bauer† attribute hardening to the presence of these strains, which may reach very large values. Internal strains can also account for a number of the changes in physical properties, particularly electrical and magnetic, and have considerable bearing on recrystallization phenomena.

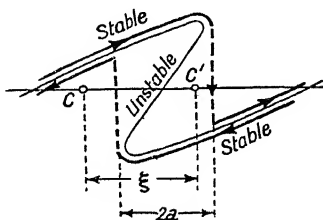


FIG. 93.

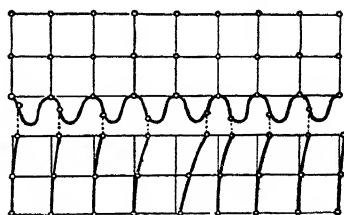


FIG. 94.

It was first shown by Czochralski,‡ and Beck and Polanyi,§ and has recently been confirmed by von Arkel and von Bruggen|| that if a single crystal is bent and then bent back, the crystals produced on annealing are larger than if the same amount of work was done all in one direction. This can only mean that some of the strain set up by the first deformation is released when the metal is bent in the reverse direction.

Certain aspects of the problem of deformation remain to be referred to.

It has been assumed in all the theories relating to hardening, etc., that the crystal structure is very much broken up in the process of deformation, and the X-ray investigations seem to bear this out. On the other hand, the direction of slip in the slip-plane is maintained throughout the extension. We can only explain this, if we assume that slip only takes place up to a certain point on each plane and that new planes are constantly being used. This is in accordance with the facts and also with Gough's idea that slip takes place first on the less perfect planes and as these become 'used' the more perfect ones,

† E. Heyn, O. Bauer, loc. cit.

‡ Czochralski, *Proc. 1st Intern. Congr. Appl. Mech.* Delft, 1924.

§ P. Beck, M. Polanyi, *Naturwiss.* **19** (1931), 505.

|| Von Arkel, von Bruggen, *Z. Physik*, **80** (1933), 804.

with cohesion approaching the theoretical values, begin to function. There is no explanation why slip should stop on a given plane unless it automatically does so when the rows of atoms on each side of the plane of partition become out of alignment, in accordance with Jeffries' theory. In this connexion there is a further difficulty. Many crystals, particularly alloy crystals, continue to deform by slip on the first octahedral plane after they have reached an orientation in which a second octahedral plane is equally inclined to the axis. If the hardening were equal on all similar planes, we should expect double slipping to occur at the point when both planes were symmetrical to the axis. Although this usually happens, there is always a tendency to go beyond this point before double slipping begins. Hence, it appears that the planes crossing the slip-plane are strengthened more than the slip-plane itself. This is in accordance with what might be expected if a rotation similar to that described by Taylor and Yamaguchi actually takes place. Certain alloy crystals never slip simultaneously on two planes, i.e. one functions at a time regardless of the fact that the shear stress on a second plane may be greater. Three stages have been observed, of which one and three concern the first octahedral plane, and two the second plane. Here is further evidence with regard to differential hardening of the planes, but any distortion that takes place during the first and second stages does not break up the first slip-plane, which is able to take over the deformation after an extension of nearly 100 per cent. This does not seem to be compatible with rotation and fragmentation of particles on the slip-plane.

The formation of twins in zinc crystals is another example of the apparent maintenance of the crystal structure even after severe distortion. Twinning has been observed to occur with crystallographic precision after a crystal has extended by slip on the basal plane to several hundred per cent.† It has been stated that zinc can deform and harden without any distortion being indicated by means of X-rays,‡ but even crystals which do show distorted X-ray reflections twin quite readily. One is therefore forced to the conclusion that whatever the distortion may be, it is of a uniform character and does not entirely destroy the symmetry of the crystal.

The increase in resistance to deformation imparted to a crystal by

† See Mark, Polanyi, Schmid, Chapter II. E. N. da C. Andrade, *J. Inst. Metals*, **48** (1932), 250.

‡ M. Straumanis, *Z. Krist.* **83** (1932), 29.

the presence of foreign atoms is attributed to the internal strains set up in the lattice by the different atomic volume of the atoms and by the irregularities caused by them in the slip-planes. If this were true, ductility would always be reduced by alloying and this is not so, although in most instances alloys are both harder and less ductile than the pure metals. Moreover, alloy crystals normally give perfect X-ray reflections so that whatever distortion there may be, it is not of the same nature as that associated with plastic deformation. It seems possible that the attraction between the two kinds of atoms is greater than between like atoms and that this is the explanation of some of the properties of alloys.

At the present time there is no satisfactory explanation of the change of properties accompanying plastic deformation, although some of them may be partially explained by reference to the presence of internal strains. But this is a very vague expression and can only be regarded as an approximation. Tammann maintains that the distortion exists in the atoms themselves, and as the problem of plastic deformation is part of the more general one of cohesion, it seems likely that the structures of atoms and the action of inter-atomic forces will ultimately provide the true explanation.

The maximum effect is usually produced during the early stages of deformation, i.e. it is associated with the initial break-down of the crystal structure. This is evident from the stress-strain diagram. A point is reached when the rate of increase in strength with elongation is very slow compared with the earlier stages of the extension and most of the other physical properties show a definite progressive diminution in the rate of change. It seems probable that very small displacements of the atoms in a lattice, whatever the cause, which may be purely mechanical or due to the presence of impurities, have a very much greater influence on the properties of the crystal than relatively larger movements at a later stage.

APPENDIX

SINCE writing the above, two papers have appeared simultaneously, both of which attempt to define with greater precision the nature of the hypothetical discontinuities in a crystal. Taylor's† 'dislocations' are very similar in character to those of Dehlinger and Prandtl and can travel along a slip-plane in much the same way as these workers have suggested. He assumes that they are present in all crystals and move under the influence of thermal agitation. They can be made to move at lower temperatures by the application of very small shear stresses, but are stopped by faults in the structure. This leads to the conclusion that the more perfect the crystal, the lower its resistance to deformation. If the dislocations have the properties assigned to them by Taylor, the stress-strain curve for a pure metal should be a parabola, and he shows that this is approximately true in fact.

Polanyi's‡ ideas are very similar in many respects. According to him, the discontinuities are not already present in a crystal before deformation begins, but arise during the process, and he quotes the difference between the plasticity of wet and dry rock-salt as confirmation. The passage of a dislocation takes place in the same way as Taylor suggests. If it passes across a whole section of a glide-plane, it leaves the structure undisturbed. Polanyi falls back on some form of distortion of the crystal structure as an explanation of the increased resistance to deformation.

† G. I. Taylor, *Proc. Roy. Soc. A*, **145** (1934), 362.

‡ M. Polanyi, *Z. Physik*, **89** (1934), 660. See also E. Orowan *Z. Physik*, **89** (1934), 614.

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